Centripetal Acceleration and Newtonian Gravity

Reminders: 24 Oct, Exam 2!
Same rooms as last time.

Today

Centripetal Acceleration
Acceleration that pulls toward the center of any curve.

Newtonian Gravity
An OK approximation for how gravity works.

A rap about your physics misconceptions.

Do you remember the conceptual pre-test? This lecture covers the stuff where you guys were the wrong-est.
You’re looking down at a frictionless channel anchored to a frictionless horizontal table top. Ignore air resistance. A ball is shot at high speed into the channel at P and exits at R.

Not a clicker (yet)

Consider the following forces:
1. downward force of gravity.
2. a force exerted by the channel, in the direction from O to Q.
3. a force in the direction of motion.
4. a force pointing from O to Q.

Which of the above forces is (are) acting on the ball when it is at point Q?

A. 1 only.  B. 1 and 2.  C. 1 and 3.  D. 1, 2, and 3.  E. 1, 3, and 4.

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Uniform Circular Motion

Remember Newton’s first law of motion?

Things will travel in the same direction at the same speed unless an external force is applied to the object.

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Uniform Circular Motion

Remember Newton’s first law of motion?

Constant angular velocity, \( \omega \) = \( \text{constant} \)

The same direction at the same speed, unless an external force is applied.

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Uniform Circular Motion

Remember Newton’s first law of motion?

Spinning a ball on a string at constant angular velocity \( \omega \) (\( \omega \neq 0 \)).

Velocity direction changes, but speed (magnitude of velocity) does not change!

WHAT FORCES ARE ACTING ON THE BALL?
Uniform Circular Motion
Remember Newton’s first law of motion?

\[ \Delta \theta \]

String pulls ball inward:
This causes
**centripetal acceleration!**

There is a centripetal force/acceleration on anything moving in a circular path!

Centripetal acceleration

\[ a_c = \frac{v^2}{r} = r\omega^2 \]

Centripetal force

\[ F_c = ma_c = \frac{mv^2}{r} = mr\omega^2 \]

Tangential and Total acceleration

Previous slides were all about constant speed.
What if we now add some acceleration in spin rate?

**Tangential acceleration!**

Instantaneous tangential velocity vector for this motion shown in green.

Tangential and Total acceleration

Previous slides were all about constant speed. What if we now add some change (acceleration) in spin rate?

Recall from last lecture the relation:

\[ a_t = r \alpha \]
Let's try it.
A race car accelerates uniformly from a speed of 40 m/s to a speed of 60 m/s in 5s while travelling counterclockwise around a circular track of radius 400m. When the car reaches a speed of 50 m/s, find

a) the magnitude of the car’s centripetal acceleration.
b) the angular speed.
c) the magnitude of the tangential acceleration.
d) the magnitude of the total acceleration.

\[
\begin{align*}
v &= v_0 + at \\
\omega &= \omega_0 + at \\
\Delta \theta &= \omega_0 t + \frac{1}{2} \alpha t^2 \\
\frac{\Delta v}{\Delta t} &= \omega + \frac{1}{2} \alpha t \\
v^2 &= v_0^2 + 2a,\Delta x \\
\omega^2 &= \omega_0^2 + 2\alpha,\Delta \theta \\
|\mathbf{a}_n| &= \sqrt{a_t^2 + a_n^2}
\end{align*}
\]

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Consider the following **forces**: 1. downward force of gravity. 2. a force exerted by the channel, pointing from Q to O. 3. a force in the direction of motion. 4. a force pointing from O to Q.

Which of the above forces is (are) acting on the ball when it is at point Q?

A. 1 only.  
B. 1 and 2.  
C. 1 and 3.  
D. 1, 2, and 3.  
E. 1, 3, and 4.

Centrifugal Force  
(fictitious force!)

Car turns into your body, which is trying to move in a straight line (it’s not that you’re actually pushed outward by something)!

Artificial gravity
Artificial Gravity

Astronauts spending lengthy periods of time in space experience negative effects due to weightlessness, such as weakening of muscle tissue. In order to simulate gravity, how many revolutions per minute would be required to create a normal force equal in magnitude to the astronaut’s weight?

\[ a_c = \frac{v^2}{r} = \text{rev} \]

Gravity as a Centripetal Acceleration

A sample of blood is placed in a centrifuge of radius 16.0 cm. The mass of a red blood cell is 3.0 x 10^-11 kg, and the magnitude of the force acting on it as it settles out of the plasma is 4.0 x 10^-11 N. At how many revolutions per second should the centrifuge be operated?

Newtonian Gravity

\[ F_G = G \frac{m_1 m_2}{r^2} \]

\( r = \) distance between centers of two objects

\( G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2 \) (Gravitational Constant)
**Newtonian Gravity**

\[ F_G = G \frac{m_1 m_2}{r^2} \]

- \( r \) = distance between centers of two objects
- \( G = 6.67 \times 10^{-11} \) N m\(^2\) / kg\(^2\)
  
  (Gravitational Constant)

Sun’s position relative to solar system center over last 50 years

**We are all gravitationally attractive!**

You’re sitting 0.5 m away from your special someone. Estimate the gravitational attraction between you.

\[ F = G \frac{m_1 m_2}{r^2} \]

- \( m_{\text{you}} = 80 \text{ kg} \)
- \( m_{\text{sp}} = 70 \text{ kg} \)

\( G = 6.67 \times 10^{-11} \) N m\(^2\) / kg\(^2\)
  
  (Gravitational Constant)

**Note: who is more gravitationally attractive?**