Pulsar Lecture Slides

Minimum Magnetic Field Strength (at surface of pulsar)

- $\mathbf{P_{rad}}$ (power radiated by magnetic dipole)
- $\mathbf{-\dot{E}}$ (rate of loss of rotational energy)
- Assuming all loss of rotational energy goes into magnetic dipole radiation:

$$P_{rad} = -\dot{E}$$

Minimum Magnetic Field Strength

$$P_{rad} = -\dot{E}$$

- We have expressions for each of these- plug in:

$$\frac{2}{3c^3}(BR^3 \sin\alpha)^2 \left(\frac{4\pi^2}{P^2}\right)^2 = \frac{4\pi^2 I\dot{P}}{P^3}$$

- Everything is a constant/feature of pulsar, except for B- solve for B:

Minimum Magnetic Field Strength

- Solving for B, we get

$$B = \left(\frac{3c^3 I}{8\pi^2 R^6 \sin^2 \alpha} \right)^{\frac{1}{2}} (P\dot{P})^{\frac{1}{2}}$$

- However, sin^2 ranges between 0 and 1, so

$$B > \left(\frac{3c^3 I}{8\pi^2 R^6} \right)^{\frac{1}{2}} (P\dot{P})^{\frac{1}{2}}$$

- Plugging in the values for a canonical pulsar (R~10^6 cm, I ~ 10^45 g/cm^2) we get

$$\left(\frac{B}{gauss} \right) > \left(3.2 \times 10^{19} \right) \left(\frac{P\dot{P}}{s} \right)^{\frac{1}{2}}$$

Characteristic Age

- We want an approximation of the age of a pulsar.
- We can get one by integrating $P\dot{P}$ over the pulsar's lifetime.
- Rearranging our magnetic field expression for $P\dot{P}$, we get

$$P\dot{P} = \frac{8\pi^2 R^6 (B \sin \alpha)^2}{3c^3 I}$$

- This is a constant, as long as B is constant in time!
- …..But is it?

Is B Constant in Time?

For a canonical pulsar, $P \approx 0.1s, \dot{P} \approx 10^{-15}\frac{s}{s}, \ddot{P} \approx 10^{-29}\frac{s}{s^2}$

a) What is the minimum magnetic field at the surface?

b) Calculate the rate of change of the magnetic field. Is it a reasonable approximation that the magnetic field is constant?

Solution

a) $B = (3.2 \times 10^{19}) \times ((0.1)(10^{-15}))^{1/2} = 3.2 \times 10^{11} gauss$

b) $$\frac{\partial B}{\partial t} = (1.6 \times 10^{19}) \times (P\dot{P})^{-1/2} \times (P\ddot{P} + \dot{P}^2)$$

$$\frac{\partial B}{\partial t} = 0.00176 \ \frac{gauss}{s}$$

Characteristic Age

- So, B is mostly constant in time, and $P\dot{P}$ is as well.
- Get an expression for **τ** using a trick:

$$P\dot{P}dt = P(\dot{P}dt) = PdP$$

- Integrate both sides over the lifetime of the pulsar:

From
t = 0 (birth)
to
t = τ (current age)

$$\int_0^{\tau} P\dot{P}dt = \int_{P_0}^{P} PdP$$

From
P = P0 (period at birth)
to
P = P (current period)

Characteristic Age

$$P\dot{P}\int_0^\tau dt = \int_{P_0}^{P} P dP$$

$$P\dot{P}\tau = \frac{1}{2}(P^2 - P_0^2)$$

$$\tau = \frac{P}{2\dot{P}}$$

ASSUMPTIONS

- B is constant in time
- P0 << P

Characteristic Age

- Is the approximation of τ more accurate for young pulsars or old pulsars?
  - Old: we assumed P0 << P.
  - Old pulsars have had more time to spin down, period has changed more since birth

Braking Index

ERA 6.1.8

- Back to $P_{rad} = -\dot{E}$
- Using different forms (angular velocity, not period) and rearranging, get expected proportionality: $\dot{\Omega} \propto \Omega^3$
- However, this is a theoretical value, and our observations might not match up: $\dot{\Omega} \propto \Omega^n$
- Braking index n gives evolution of pulsar spin over time. Observed values ranging from 1.4 to 3

Braking Index

- Why the discrepancy?
- We assumed that $P_{rad} = -\dot{E}$: very simplistic model of pulsar radiation
  - Magnetosphere- plasma surrounding pulsar
  - Changing magnetic field over time
  - Timing noise & glitches

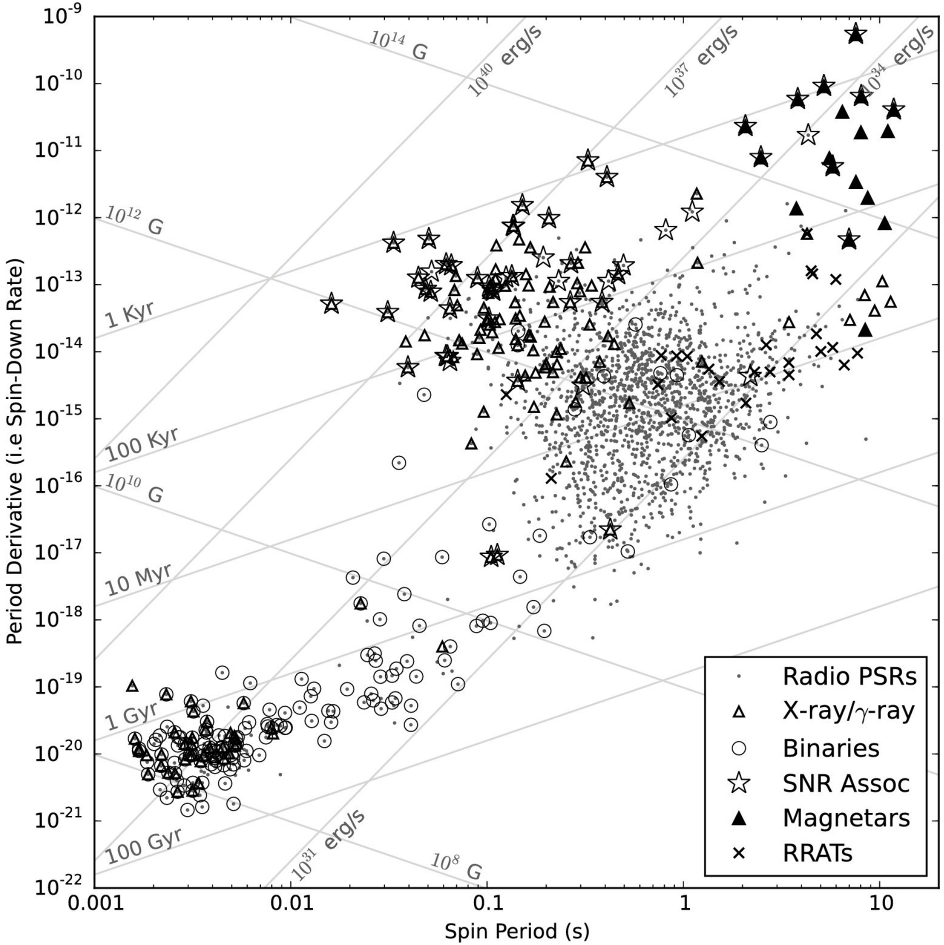
Braking Index

- Starting from $\dot{\Omega} \propto \Omega^n$

$$n = \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} = 2 - \frac{P \ddot{P}}{\dot{P}^2}$$

Lives of Pulsars

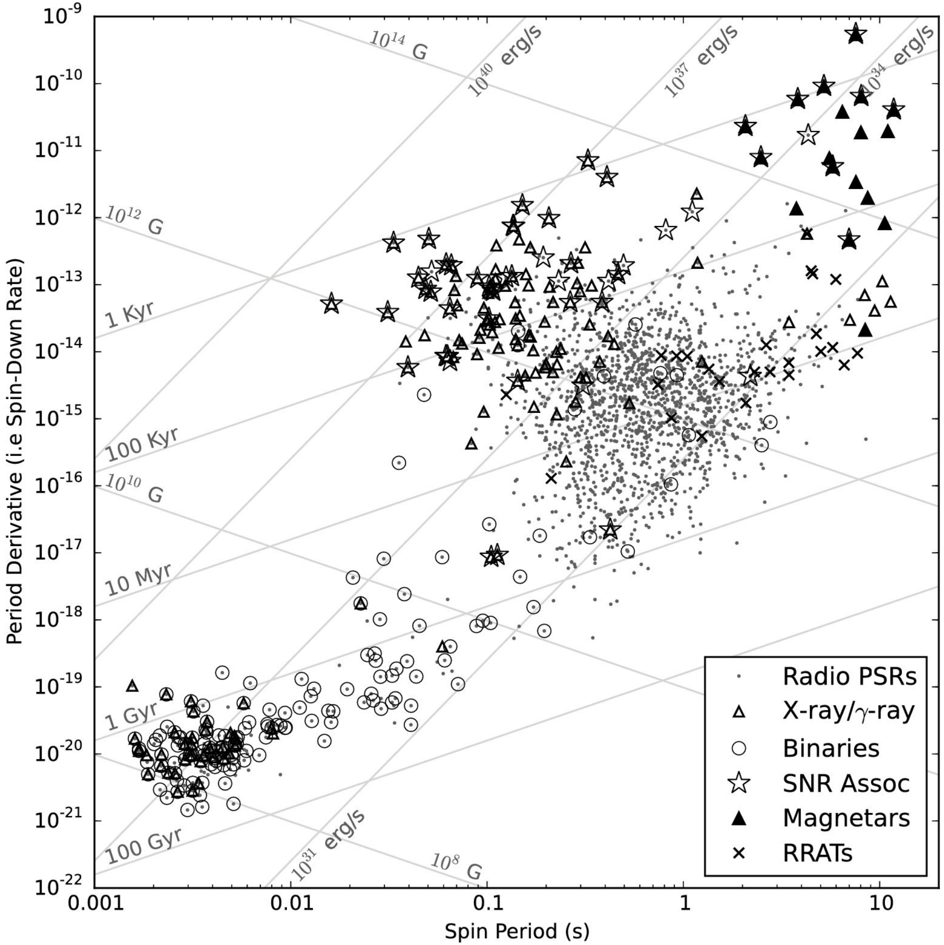
- -Ė, Bmin, **τ**, all determined by location on diagram
- Different populations
- Lines of constant Bmin, -Ė, **τ**



$P - \dot{P}$ diagram

Lives of Pulsars

- How will pulsar evolution over time look on the diagram?



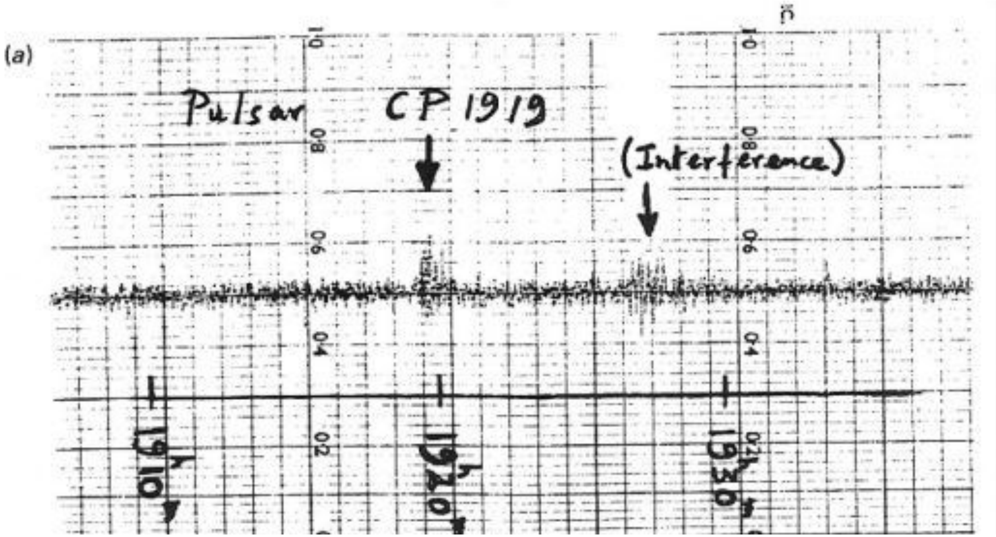
$P - \dot{P}$ diagram

Pulsar Astronomy

1. Detection
2. Timing
3. Science!

Detection

- Pulsars are weak and inconsistent
- Searching for pulsars- can't always just point a telescope and be sure you'll see it
- How to find one, then?



Detection

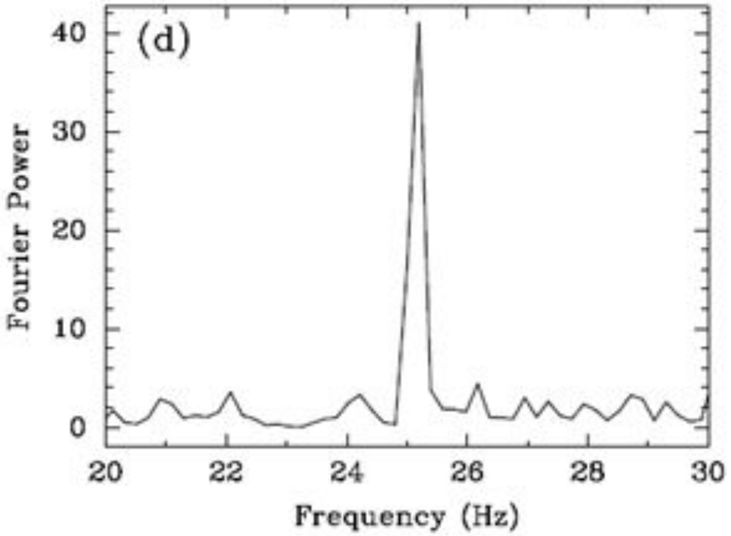
- Pulsar signal should be dispersed and periodic
- Dispersion- low frequency part of signal moves slower than high frequency part, because of ISM

Detection

- Pulsar signal should be dispersed and periodic
- Dispersion- low frequency part of signal moves slower than high frequency part, because of ISM
- Search for periodic signals
  - Take FT of data (which should be periodic)
  - FT of periodic function (ie sin/cos)?

Detection

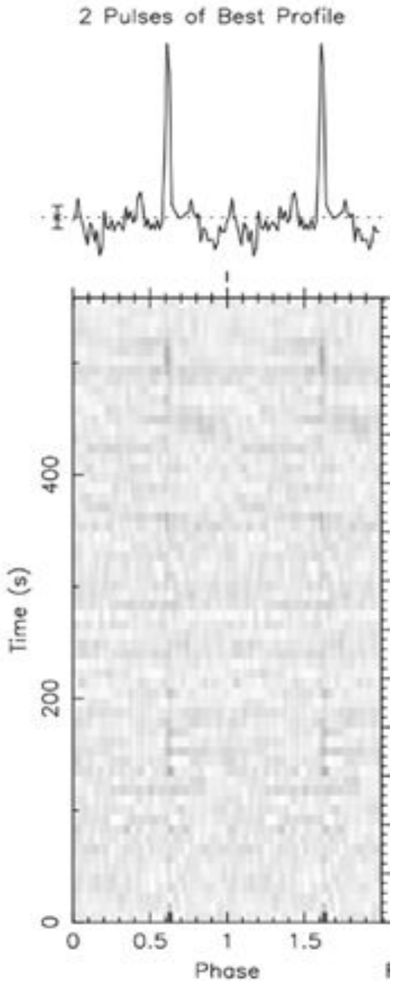
- Search for periodic signals
  - Take FT of data
  - Not quite delta function, but peak at good approx. of frequency/period



- Signal is still weak- how do we boost our S/N…?

Detection

- "Fold" data at best period
- Pulse should arrive at the same point in the rotation every time (over short timescales)
- Signal adds up more quickly than noise

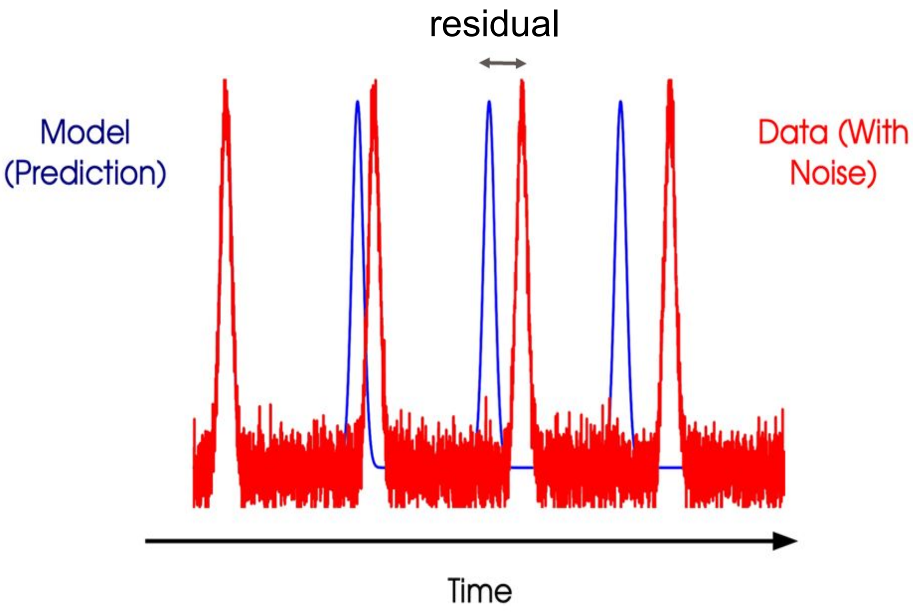


Timing

- Get a good period at one small chunk of time
  - For accurate science, need extremely precise values of P and its derivatives at all times
  - As well as other aspects- RA, Dec, DM, orbital parameters for binary systems

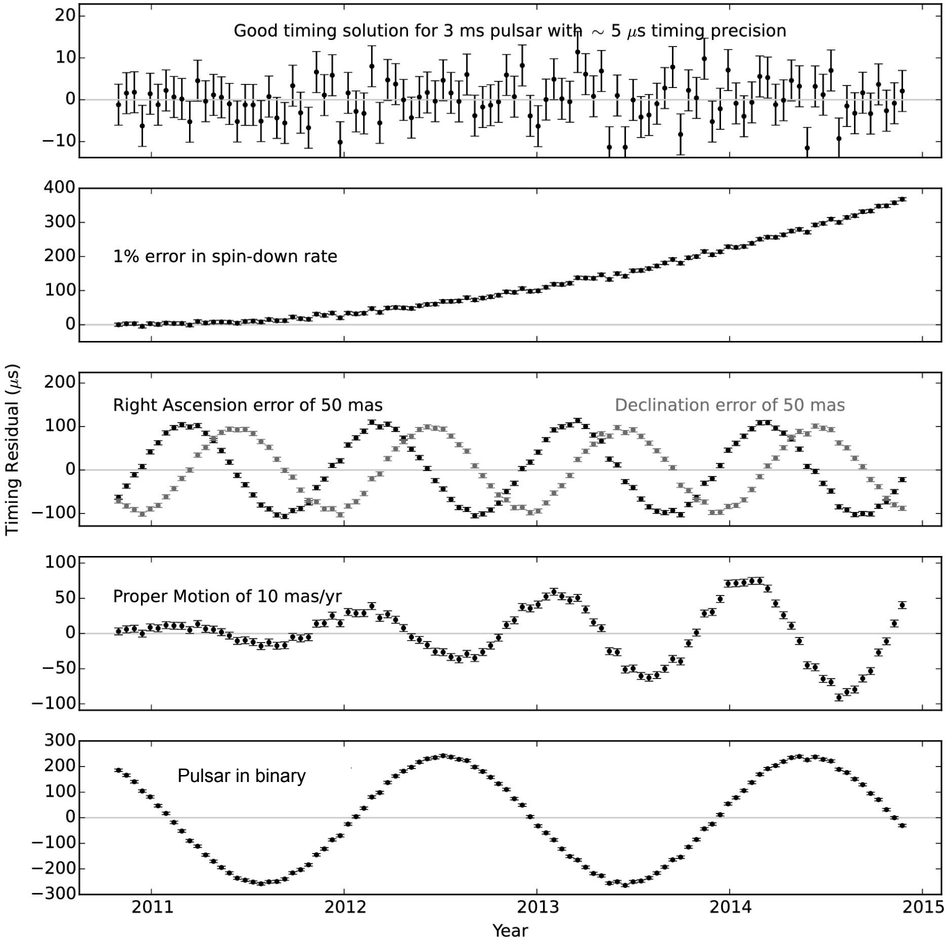
Timing

- Obtain *times of arrival* (TOAs)- at what precise time did pulse arrive at telescope
  - Compare model of when you think it'll arrive vs. when it actually does (residuals)
  - By adding more to your model, you want to minimize residuals- minimize error in model



Timing

- If your model is good, your residuals will be randomly distributed around 0
- If not, tweak parameters of your model to get better residuals, rinse and repeat



Why Pulsars?

- Extreme physics- i.e. densities and magnetic fields
- Gravitational wave detection
  - Array of well-timed pulsars across sky
- Tests of GR
  - Binary pulsar system- loses energy to gravitational radiation; period decreases
- Also: electron density of the galaxy, plasma physics, radio emission mechanisms, extremely dense matter, high-precision timing, data analysis, signal processing….