Radio Astronomy (ASTR700) Problem Set #5

Due March 10!

Interferometry

Disclaimer: This homework tests the theory, “short questions tend to be hard, while long questions tend to be easy.” So, enjoy the word-wall below you, which hopefully represents easy but informative questions.

Second Disclaimer: This homework is also meant to get you using and looking under the hood of at least one online tool commonly used for telescope scheduling.

1. (10 points) ERA Equation 3.203 gives the rms noise in units of point-source flux density for an interferometer image made with natural weighting, a single polarization, and a perfect analog correlator:

$$
\sigma_s = \frac{2kT_{\text{sys}}}{A_e[N(N-1)\Delta\nu \tau]^{0.5}}
$$

Most VLA observations use two polarizations and there is some signal loss due to a “correlator efficiency” term; this is common to antennae in the A/D conversion from analog signal to specific sampling levels (usually 1-, 2-, 3-, or 8-bit sampling). The correlator efficiency of VLA’s 8-bit sampler is $$\eta_c \approx 0.93$$, thus a better estimate of the VLA noise is

$$
\sigma_s = \frac{2kT_{\text{sys}}}{\eta_c A_e[n_{\text{pol}}N(N-1)\Delta\nu \tau]^{0.5}}
$$

VLA’s observations typically include $$N = 25$$ antennae (of 27, some tend to be down for maintenance), each of which have a diameter of 25 m. At $$\nu = 3 \text{ GHz}$$ (“S-band”), the system noise temperature is about 40 K and the aperture efficiency is $$\eta_a \approx 0.62$$. The bandwidth of the receiver is 2 GHz, but there are GPS downlinks in the band that limit the maximum usable bandwidth to around $$\Delta\nu \approx 1.5 \text{ GHz}$$. Using these parameters, estimate how much on-source integration time is needed to reach an RMS noise of 10 $$\mu\text{Jy/beam}$$.

Answer: Using the second equation above and plugging in our knowns:

$$
T_{\text{sys}} = 40 \text{ K} \quad (1)
$$

$$
\eta_c = 0.93 \quad (2)
$$

$$
A_e = \eta_a A_{\text{geom}} = (0.62)(\pi)(25/2)^2 = 304.3 \text{ m}^2 \quad (3)
$$

$$
n_{\text{pol}} = 2 \quad (4)
$$

$$
N = 25 \quad (5)
$$
\[ \Delta \nu = 1.5 \times 10^9 \text{ Hz} \quad (6) \]
\[ \sigma_s = 10 \ \mu\text{Jy/beam} \quad (7) \]

All we have to do here is solve for \( \tau \) and plug the numbers in. Thus

\[ \tau = \left( \frac{2kT_s}{\sigma_s\eta_cA_e\sqrt{n_{\text{pol}}N(N-1)\Delta \nu}} \right)^2 \]

and

\[ \tau = \left( \frac{(2)(1.38 \times 10^{-23})(40)}{(10 \times 10^{-6})(10^{-26})(0.93)(304.3)\sqrt{(2)(25)(24)(1.5 \times 10^9)}} \right)^2 \]

\[ \tau \approx 845 \text{ s} \]

Or around 14 minutes, 5 seconds.

2. (25 points) The VLA Exposure Calculator Tool at [https://obs.vla.nrao.edu/ect](https://obs.vla.nrao.edu/ect) is a handy tool to help you prepare observations by estimating the sensitivity given a particular observing length, or alternately estimating the required on-source time to reach some sensitivity limit. It also provides live feedback on whether your observation will hit the confusion limit or have other various issues.

Use this calculator to check your result from the previous question. Enter the relevant values: 3.0 GHz for the Representative Frequency and 1.5 GHz for the Bandwidth (Frequency). Then set Array Configuration=A, Number of Antennas=25, Polarization Setup=Dual, Type of Image Weighting=Natural, Digital Samplers=8 bit, Elevation=Medium, Average Weather=Summer, Calculation Type=Time, and RMS Noise (units/beam)=10 \( \mu\text{Jy} \). Do the following exercises:

(a) Convert the result for Time on Source (UT) to seconds. What value did you get?
[Hint: It should equal within a few percent the result you calculated for question 2. If so, you know more about calculating sensitivities than many NRAO users. If not, check that all of the input fields for the VLA Exposure Calculator are correct. If they are, recheck your calculation for question 2.]

\[ \text{Answer:} \]
I got 13 m 56 s. This is 836 seconds, or about 99\% of the answer from problem 1. Pretty close!

(b) Change the Array Configuration from A \( (B_{\text{max}} \sim 35 \text{ km}) \) to C \( (B_{\text{max}} \sim 2.4 \text{ km}) \) and observe the change in the required Time on Source (UT). It should increase somewhat. Detail and justify the reason that required observing time might
increase at this C array configuration. If you can’t think of why, some light might be shed on taking a more thorough look at all the things reported by the Exposure Calculator Tool.

Answer:
The larger synthesized beam in C-configuration (a more compact configuration) leads to a larger value for the confusion limit. However, because $\sigma_c$ is sufficiently small that we can still lower $\sigma_s$ without reaching the hard confusion limit, to reach our desired sensitivity we need to simply observe a little bit longer.
The story would be different if $\sigma_c$ were really large. In D configuration, we are unable to reach our desired sensitivity because of confusion!

(c) So far, you’ve calculated the integration times needed to reach point-source-sensitivity RMS noise $\sigma_S = 10\mu$Jy. Next, set RMS Brightness (temp) to 1K and find the Time on Source (UT) values needed to reach a brightness noise level $\sigma_T = 1$K in the A and B configurations. You should find that it takes about $100 \times$ longer to reach $\sigma_T = 1$K in the A configuration (28m 14s) than in the B configuration (15s). To image an extended source having a given brightness temperature, it takes a lot longer to make a higher-resolution image. Derive an equation showing how $\tau$ scales with the Approximate Beam Size $\theta$ for $\sigma_T = 1$K in the A and B configurations, and verify that your equation agrees with the above times from the VLA Exposure Calculator.

[Another side note: You can produce and print your results by using the “save” input at the bottom of the screen to produce a PDF; you don’t need to turn these in but doing this is required when you’re proposing for VLA observations!]

Answer: This question was meant to make you think through spectral brightness; this is discussed in ERA 3.7.6. You can think of this as the fact that interferometers measure some flux density per synthesized beamwidth (hence the oft-quoted units Jy/beam). Because $S_\nu = \int I_\nu \Omega$, the quoted Jy/beam can be thought of as a measure of flux or a brightness—to consider brightness, you only need to consider the actual size of the beam. To get the actual flux density of the source, you have to integrate over some solid angle of the source.

A matter of practicality: in common software packages like CASA, this Jy/beam measure is typically called the “peak flux”, whereas if you integrate over some region to get units of Jy, it is typically called the “integrated flux.”

In the Jy/beam units, the value of the flux density equals the value for brightness, and so it is a convenient measure when considering integrating the flux (or brightness) over the solid angle of the source for some number of synthesized beamwidths across the full extent of the source.
That said, ERA gives a brief derivation to arrive at surface brightness sensitivity in terms of beam size, which for a fixed value of $\sigma_s$ gives:

$$\sigma_T \propto 1/\Omega_A \propto \theta^{-2}$$

(Equations 3.204 and 3.205 in ERA). We also know from the radiometer equation that $\tau \propto \sigma_s^{-2}$, so we know that for a fixed sensitivity limit (in this case 1K), $\tau \propto \theta^{-4}$. To check this, I checked A and B configuration and found $\tau_A = 1694$ s and $\theta_A \simeq 0.977''$. In B config I found $\tau_A = 15$ s and $\theta_A \simeq 3.203''$. Considering that $\tau \theta^4$ is independent of $\theta$, and thus

$$\frac{\tau_A \theta_A^4}{\tau_B \theta_B^4} = 1$$

and putting in the ECT values,

$$\frac{(1694)(0.977)^4}{(15)(3.203)^4} \simeq \frac{1543}{1579} = 0.98 \simeq 1$$

Quite close.