Riding the (conceptual) pulse train: a thorough think-through of convolution.
In our discussion of convolution from the previous class, we saw that a kernel function $g(x)$ convolved with a Dirac delta function $\delta(x)$ reproduced the kernel, i.e.

$$g(x) \otimes \delta(x) = g(x)$$

Where the delta function can be stated as an "infinite spike of unit area"; that is, it is infinity at a single value. Doodle the delta function.

Now consider the Comb function, also referred to as the "Shah" function, which can be thought of as a series of regularly-spaced delta functions:

$$\mathrm{III}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$$

Doodle the Shah function.

There are a few interesting properties of the Shah function. First, the Fourier transform of the Shah function is such that $\mathrm{III}(x) \rightleftharpoons \mathrm{III}(s)$. Second, similar to the delta function, the Shah function will serve to produce **replication** of a kernel g(x):

$$g(x) \otimes \mathrm{III}(x) = \sum_{n=-\infty}^{\infty} g(x-n)$$

Draw some arbitrary function $g(x)$, and then draw $g(x) \otimes \mathrm{III}(x)$. Note, this will be easiest to conceptualize if g(x) follows ($\lim\, |x| \to \infty = 0$),

Finally, one can think of $\mathrm{III}(x)$ as a **sampling function** if multiplied by (instead of convolved with) function g(x). Draw a doodle of this multiplication of $g(x)$ and $\mathrm{III}(x)$:

$$g(x)\ \mathrm{III}(x) = \sum_{n=-\infty}^{\infty} g(n)\delta(x-n)$$

OK, I think you are ready for the actual question now; let's put the Shah function to use. Consider a pulsar: it can be thought of as a train of boxcar functions. How can this be expressed as a combination of $\sqcap(x)$ and $\mathrm{III}(x)$? Write the expression:

Finally, what is the Fourier transform of this pulse train? Express this mathematically OR graphically (whichever you find easier).

Microscopic systems: Limitations of the Larmor Formula we derived in class.
In class, we derived the Larmor formula (shown here in CGS):

$$P = \frac{2}{3}\frac{q^2\dot{v}^2}{c^3}$$

This formula will be important for a number of derivations in future lectures in this class, but a critical thing to recognize about this is that 1) it applies ONLY to non-relativistic particles (i.e. velocities must be small), and 2) it does not contain quantum mechanical constraints, therefore only applies for macroscopic systems (see rather extensive alternate QM derivations elsewhere). That is, this formula should not be applied, except with great caution, to atomic-level systems. This problem highlights the latter issue.

The classical Larmor's formula for radiation from an accelerated charged particle implies that the electron orbiting a proton in a hydrogen atom will radiate away its kinetic energy in a small fraction of a second, and the atom will collapse. This striking failure was one of the problems in classical physics that led to the development of quantum mechanics. A classical hydrogen atom consists of an electron in a circular orbit around a proton, with the centrifugal force:

$$F_c = \frac{m_e v_e^2}{r_0}$$

which balances the Coulomb force, e^2/r_0^2, where $r_0 \sim 5 \times 10^{-9}$ cm is the Bohr radius. Classically, the radiative lifetime t of this atom is the electron kinetic energy E divided by the Larmor power radiated at a radius $r = r_0$. Estimate t.

Exercising our theorems

Show that the Fourier transform of a triangle function is a sinc^2 function. The trick is this: you must demonstrate this using only theorems and derivations we saw in our previous class!

As a reminder you also saw this, which is information you can use: