ASTR469: Problem Solving Day #1.

Order-of-magnitude approximation is always valid!

1. **Understanding frequency-dependent appearance of hot-bodied objects.** Compute the spectral brightness (I_ν), spectral flux, and spectral luminosity of the Sun in the radio band, using an observing frequency of 10 GHz.

Assume sun is a BB, so $I_\nu = B_\nu$. 10GHz emission, so Rayleigh-Jeans approx is valid!

$T = 5800\text{K}$

$\nu = 10\text{GHz}$

$c \simeq 3\times10^8 \text{ m/s}$

$k \simeq 10^{-23} \text{ J/K}$

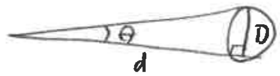
☆ $I_\nu = B_\nu = \frac{2kT\nu^2}{c^2} = \frac{2(10^{-23})(5800)(10^{10})^2}{(3\times10^8)^2} \simeq \boxed{10^{-16} \frac{W}{m^2 Hz\, sr}}$ I_ν

☆ Flux: $F_\nu = \int I_\nu \cos\theta \, d\Omega$

assume looking @ target, so $\cos\theta \to 1$

$F_\nu \simeq I_\nu \Omega_\odot$

what is solid angle of sun?



Small angle approx:

$d_{sun} \simeq 1.5\times10^{11} m$

$D_{sun} \simeq 10^9 m$

$\tan(\theta) \simeq \frac{D}{d}$

Small angle approx: $\tan\theta \simeq \theta \simeq \frac{D}{d} \simeq 10^{-2}$

$\Omega \simeq \pi\theta^2$

officially, θ here is $\frac{1}{2}\times10^{-2}$, but with OOM I'm dropping the $\frac{1}{2}$.

$\Omega \simeq 10^{-4} sr$

$F_\nu = I_\nu \Omega_\odot \simeq (10^{-16} \frac{W}{m^2 Hz\, sr})(10^{-4} sr) \simeq \boxed{10^{-20} \frac{W}{Hz\, m^2}}$ F_ν

☆ Luminosity.

$L_\nu = 4\pi d_\odot^2 F_\nu \simeq (4)(3)(10^{11})^2(10^{-20})$

$(10)(10^{22})(10^{-20})$

$\boxed{\simeq 1000 \frac{W}{Hz} \quad L_\nu}$

$A_{det} = 3\,m^2$
$\lambda = 500 \times 10^{-9}\,m$
$N_{phot\ need} = 10^4$

$F_E = 1000\,\frac{W}{m^2}$

2. **Relating flux and luminosity to an observation.**
a) You have an optical telescope with a $3\,\text{m}^2$ collecting area that operates in the 500 nm waveband. The noise in this detector is such that you must catch at least 10^4 photons to detect an object. **How long will it take you to detect the Sun?** (Useful number: the solar flux reaching Earth, integrated over your waveband, is around $1000\,\text{W}\,\text{m}^{-2}$.) Note: assume that there are no losses in the space between your detector and the targets, and no other losses due to filters or imperfections in the telescope itself (in reality, absorption and scattering in the atmosphere and clouds can absorb 15-80% of the Sun's light at this wavelength!).

b) Bonus if you get the previous one done quickly:
How far away could you detect an object of the same luminosity of the Sun with the same requirement for detected photons? For this question, assume you will take a 2-minute exposure (i.e. you'll collect light for a total of 2 minutes).

a) F_E $1000\,\frac{W}{m^2}$ is an energy flux through unit surface of $1\,m^2$.

Energy of one photon: $E = h\nu = \frac{hc}{\lambda} \simeq \frac{10^{-33} \times 10^{8}}{10^{-7}} \simeq 10^{-18}\,J$

So photon flux at Earth per $1\,m^2$ is:

$$F_{phot} = \frac{F_E}{E_{phot}} = 1000\,\frac{J}{s\,m^2} \times \frac{1\text{ photon}}{10^{-18}\,J} = 10^{21}\,\frac{\text{photons}}{s\,m^2}$$

Through detector:

$A = 3\,m^2$

$$F_{phot} \times A = \frac{10^{21}}{m^2 s} \times 3\,m^2 \simeq 10^{21}\,\frac{\text{photons}}{\text{second}}$$ through detector surface.

So if we need 10^4 photons just wait long enough:

$$\frac{10^4\text{ photons}}{10^{21}\text{ phot/s}} \simeq \boxed{10^{-17}\,s \text{ to detect Sun!}}$$

b) Use $L = 4\pi d^2 F$ to get the L of Sun in this band:

$L \simeq 4\pi (1.5 \times 10^{11} m)^2 \times (10^3 \frac{W}{m^2})$

$\simeq 10 \times 10^{22} \times 10^3$

$L \simeq 10^{26}\,W.$

And we need:

$\frac{10^4\text{ phot}}{120\text{ seconds}} \simeq$ ~~required photon flux~~ $100\,\frac{\text{phot}}{s}$

In $3\,m^2$: $100\,\frac{\text{phot}}{s} / 3\,m^2 = 30\,\frac{\text{phot}}{s\,m^2}$ Required photon flux

Make it an energy flux!

$30\,\frac{\text{phot}}{s\,m^2} \times E_{photon} = (30)(10^{-18}) = 3 \times 10^{-17}\,\frac{J}{s\,m^2}$

Now use $L = 4\pi d^2 F$ again!

$10^{26} = 4\pi d^2 (3 \times 10^{-17}) \Rightarrow \boxed{d \simeq 10^{21}\,m}$