ASTR469: Homework #4.
Due 15 Feb at beginning of class.

1. (a-c, 1 pt each) Using the principles of refraction and reflection, draw a rough ray diagram for the following pieces of telescope apparatus. What happens to the light within the apparatus (in the case of the lens, at least) and then after it is passes the apparatus? Be at least conceptually precise (there is no need to use a protractor), and explain what should happen if you think your diagram is not clear.

\[ M_1 \sin \theta_I = M_2 \sin \theta_R \]
\[ \frac{M_1}{M_2} = \frac{\sin \theta_R}{\sin \theta_I} \]
So going small \( M \) to bigger \( M \), light bends toward surface normal. Going big \( M \) to small \( M \) (entering lens), light bends away from surface normal.
So this is a dispersive, rather than focusing lens.

(d) (2 pt) Approximate the f-number of the mirrors in (b) and (c).

(e) (2 pt) Assume a lens of 24-inch diameter with the same f-number as (c)'s mirror is used in conjunction with an eyepiece to allow people to view the sky. If you wanted to achieve a magnification of 50x, what focal length would be required for your eyepiece? Express your answer in millimeters.
2. a) (3 pt) By what fraction does the number of photons received per second from a 10th magnitude star differ compared to that of a 9th magnitude star?

b) (4 pt) Assume you have two different telescopes, each observing one of the previous stars. To collect the same number of photons from each star per second, which star should the larger telescope be observing? How many times larger must the larger telescope be? [Note: if you didn’t get (a), just assume a ratio and use that to complete this question.]

3. (3 pt) Most of the best optical telescope designs on Earth are seeing-limited. You’re picking a telescope for an observing site that has a typical seeing of 1”. Given that you’ll be doing observations in the optical band, and you want to at least reach a resolution that matches your seeing limit, what is the biggest diameter you should make your primary mirror? Choose a representative wavelength in the middle of the optical band.

4. Compute the diffraction limit in arcseconds using the Rayleigh criterion for the following telescopes (1 pt each):
   
   a) Hubble (2m diameter) telescope operating at 500nm
   b) Chandra X-ray observatory (2m diameter) observing 1keV photons
   c) GBT (100m diameter) observing at 1.4 GHz
   d) Arecibo (300m diameter) observing at 5 GHz
   e) James Webb Space Telescope (6m diameter, due to launch next year) observing at 2 microns
   f) Comment on any trends you observe.

5. (Throw-back question; more coordinates practice!) The star Arcturus (J2000 RA = 14h15m39.7s, Dec = +19°10'57'') is the fourth brightest star in the sky. You’re lost at sea on a sailing ship and observed one clear night that Arcturus rose at 9:00 pm.

   a) (2 pt) If you wanted to observe this star rise again 3 days later, assuming your ship has not moved significantly, at what clock time (with precision within one minute) should you expect to see Arcturus rise?

   b) (2 pt) Over the night, Arcturus got to a maximum elevation of 40°. At what terrestrial latitude is your ship (within one degree)? [Hint: try drawing the path Arcturus would appear to take on the sky compared to the north celestial pole.]

   c) (2 pt) If Arcturus is on your meridian, then what is the hour angle of Deneb (J2000 RA = 20:40:00)?
1. See front page.

1a. I used a ruler to measure mirror diameter and focal point’s distance from surface.

\( D = 43 \text{ mm} \)
\( f = 13 \text{ mm} \)
\( N = \frac{f}{D} \approx 0.3 \)  
 nesta en yow could also approximate the ratios without a ruler.

(c) \( D = 43 \text{ mm} \)
\( f = 30 \text{ mm} \)
\( N \approx 0.7 \)

1b. \( D = 24 \text{ in} = 609.6 \text{ mm} \)

if \( N = 0.7 \), \( f = ND = 42.7 \text{ mm} \) for this mirror.

Magnification \( m = \frac{f_p}{f_e} \) \( f_e = \frac{f_p}{m} = \frac{42.7 \text{ mm}}{0.7} = 60.3 \text{ mm} \)

1c. It's asking about a photon flux:

\[ \frac{N}{s\text{ m}^2} \] - same thing as flux/energy.

Magnitudes are defined by flux ratio:

\[ \frac{F_{10}}{F_q} = 10 \]

So the photon flux ratio should equal this quantity.

\[ \frac{F_{10}}{F_q} = 10 \times 0.4(9-10) \]

\[ \frac{F_{10}}{F_q} = 0.398 \approx 0.4 \]

2a. Larger telescope should observe a fainter object: \( m = 10 \text{ star} \)

We want the same photons collected from each telescope, and know that photons collected is proportional to aperture surface area. 

\[ F \alpha A \]

So smaller telescope observing \( m = 9 \) star must have \( A_{\text{small}} = 0.4A_{\text{large}} \).
3. Seeing limit: 1"
   Optical band: $\lambda = 500 \text{ nm}$
   $\Theta_{\text{res}} = \frac{\lambda}{D}$
   $1'' \approx \frac{500 \text{ nm}}{D}$

   $1'' \times \frac{1 \text{ deg}}{3600''} \times \frac{1 \pi \text{ rad}}{180 \text{ deg}} \approx 1.3 \times 10^{-6} \text{ rad}$

   \[ D = \frac{500 \times 10^{-9} \text{ m}}{5 \times 10^{-6} \text{ rad}} = \frac{5 \times 10^{-3}}{5 \times 10^{-6}} = 0.1 \text{ m} \]

   10 cm! Bigger telescopes will not do much better for resolution due to seeing, but of course they will improve your sensitivity by collecting more photons.

4. a) Hubble
   UV/optical
   $\lambda = 500 \times 10^{-9} \text{ m}$

   $\Theta = \frac{\lambda}{D} = 2.5 \times 10^{-7} \text{ rad}$

   b) Chandra
   $D = 2 \text{ m}$

   X-ray
   $\lambda = 2.4 \times 10^{17} \text{ Hz}$

   $\Theta = 1.2 \times 10^{-4} \text{ m}$

   $\Theta = \frac{\lambda}{D} = 6 \times 10^{-10} \text{ rad}$

   c) GBT
   $D = 100 \text{ m}$

   Radio
   $\nu = 1.46 \text{ GHz}$

   $\lambda = 2.14 \times 10^{-2} \text{ m}$

   $\Theta = \frac{\lambda}{D} = 2 \times 10^{-3} \text{ rad}$

   d) Arecibo
   $D = 300 \text{ m}$

   Radio
   $\nu = 5 \text{ GHz}$

   $\lambda = 6.0 \times 10^{-2} \text{ m}$

   $\Theta = \frac{\lambda}{D} = 2 \times 10^{-4} \text{ rad}$

   e) JWST
   Optical/IR
   $D = 6 \text{ m}$

   $\lambda = 2 \times 10^{-6} \text{ m}$

   $\Theta = \frac{\lambda}{D} = 3 \times 10^{-7} \text{ rad}$

   f) Radio dishes are huge but have comparatively much less resolution! Also, optical res. is well below the seeing limit. But fortunately all these optical/X-ray scopes are in space!
5. Arcturus

RA = 14h 15m 39.7s
Dec = +19° 10' 57"

Rise at 9:00 pm (clock/watch time)

a) Solar (watch) day: 24h,
Sidereal day: 23h 56m 45s.
Star rises 4m earlier each day.
3 days later, ~12 minutes earlier.

\[ \Rightarrow 8:48 \text{ pm} \]

b) \[ \text{Dec: } 19° + \frac{10 \text{ min}}{60} + \frac{57 \text{ sec}}{3600} = 19.1825° \]

Hmmm! This doesn't seem right.
A source of Dec 19° should have gotten higher. Maybe we're in the south?

40° + \phi \approx 70°

\[ \phi \approx 30° \text{ NCP below horizon.} \]

So latitude \( \approx -30° \)

Note: could also use
\[
\sin(90°) = \sin(\phi) \sin(\delta) + \cos(\phi) \cos(\delta) \cos(\lambda)
\]
to solve for \( \phi \).

C) Arcturus @ meridian, \( \text{ST} = 14h 15m 39.7s \)
RA_doz = 20h 40m 00s

\[ \text{HA} = \text{LST} - \text{RA} \]

\[ 14h 15m - 20h 40m = -6h 25m \]

\[ \frac{14.25 - 20.67}{25.25 - 36.75} = -6.42 \]