Please return SCANTRON ONLY to TA or instructor before you leave the room.
SAVE YOUR EXAM AND MARK YOUR ANSWERS HERE TOO;
In the next few weeks, you may be given the chance to improve your score.

Your Name: Prof Sarah  TEST #3
Print clearly.

Please also write your name and “Test 3” at the bottom of the scantron.
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1. I take an orange of radius 0.03 m and roll it down a plank 5 m long. Assuming it does not slip at all, how many times does the orange roll (i.e. how many revolutions does it go through) as it travels across the whole plank?
   
   a. 9549.3 times
   b. 166.7 times
   c. 53.0 times
   *d. 26.5 times*
   e. 4.2 times

   \[ r = 0.03 \text{ m} \]
   \[ \Delta x = 5 \text{ m} \]
   \[ \Theta = \frac{\Delta x}{r} = \frac{5}{0.03} = 166.6 \text{ rad} \]

   In revolutions:
   \[ 166.6 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 26.5 \text{ rev} \]

2. I am about to race my friend in a car. As I cross the start line driving at 5 m/s, it takes me about 4 seconds to speed up to 25 m/s. What is the average angular acceleration \( \alpha \) experienced by my tires in this time? Assume my tires have a diameter of about 0.5 m.

   a. 1 rad/s²
   b. 5 rad/s²
   c. 10 rad/s²
   d. 15 rad/s²
   e. 20 rad/s²

   \[ V_0 = 5 \text{ m/s} \]
   \[ V = 25 \text{ m/s} \]
   \[ t = 4 \text{ s} \]

   \[ \alpha = ? \]

   \[ d = 0.5 \text{ m} \]
   \[ r = 0.25 \text{ m} \]

   \[ \omega = \frac{25}{0.25} = 100 \text{ rad/s} \]
   \[ \omega_0 = \frac{5}{0.25} = 20 \text{ rad/s} \]

   \[ \alpha = \frac{\omega - \omega_0}{t} = \frac{20 \text{ rad/s}}{4 \text{ s}} \]
3. What kind of acceleration does the force of gravity from the sun apply to the Earth to cause its orbit?
   a. Angular acceleration
   b. Tangential acceleration
   c. Centripetal acceleration
   d. Both (a) and (b)
   e. None of the above.

   The force is inward along the radius of the Earth's orbit.

4. A 5.0 kg mass is swinging around in a circle on a flat, frictionless tabletop. It's swinging on a string 0.1 meters long, and I've set it to be at a constant angular velocity of 7.0 rad/s. What is the tension in the string (hint: remember that the string is applying a centripetal force on the mass, accelerating it ever inward)?
   a. 3.5 N
   b. 5.0 N
   c. 12.3 N
   d. 24.5 N
   e. 49.0 N

\[
\sum F_c = ma_c \\
T = ma_c \\
d = \theta = \frac{v^2}{a_c} = 0.1(7^2) \\
T = 5(0.1)(49) = 2.45 N
\]

5. For the spinning mass in the previous problem, consider the following forces:
   - Downward force of gravity.
   - Normal force from the tabletop.
   - A force pointing radially inwards from the mass's circular motion.
   - A force pointing radially outwards from the mass's circular motion.
   - A force in the direction of motion of the mass.

Which of the above forces is (are) acting on the mass during its motion?
   a. 1 and 2 only.
   b. 1, 2, and 3.
   c. 1, 2, and 4.
   d. 1, 2, 3, and 5.
   e. All of the forces.
6. A thief (m = 80kg) has gotten through all security and finally reaches the vault shown below. The turn-wheel to open the vault has spokes of length 0.1 meters. Beyond this door lies a lifetime of fortune, but the thief did not remember to bring any tools to assist with opening the vault door. Assume that the vault opens by beginning to turn the wheel clockwise, and that the wheel will begin to turn if a torque of 78.4 N m is applied. Could the thief open the door by hanging on the wheel at the point indicated by a star in the figure (assume the hang is perpendicular to the spoke)?

   ![Diagram of vault and turn-wheel]

   - a. No.
   - b. Yes, even if there is friction in the turn-wheel.
   - c. Yes, as long as the turn-wheel is frictionless.
   - d. Not enough information to determine.

\[ F_g = mg = (80 \times 9.8) = 784 \text{ N} \]

This weight applies a perpendicular force, causing a torque on the wheel.

\[ \tau = r F = (0.1)(784) = \boxed{78.4 \text{ Nm}} \]

If there is friction, it would oppose this torque! So there must be no friction.

7. A net torque is applied to an object. What does that object experience?

   - a. A constant angular speed. (Correct answer)
   - b. An angular acceleration.
   - c. An increasing or decreasing tangential velocity.
   - d. Both (b) and (c).
   - e. None of the above.

\[ \sum \tau = I \ddot{\alpha} \]

\( \ddot{\alpha} \) will be angular accel.

\[ \alpha_T = \nu \ddot{\alpha} \]

\( \dot{\nu} \) will be tangential accel, thus a change in tangential velocity.
8. A carousel is at rest, and is switched on. The motor applies a net torque of 300 N m, and the moment of inertia of the carousel is 8000 kg m². At what rate is the carousel spinning after 5 seconds?

\[ \begin{align*}
& \text{a. } 0.19 \text{ rad/s} \\
& \text{b. } 0.38 \text{ rad/s} \\
& \text{c. } 0.76 \text{ rad/s} \\
& \text{d. } 1.52 \text{ rad/s} \\
& \text{e. } 15 \text{ rad/s}
\end{align*} \]

\[ \sum \tau = I \alpha \\
300 = 8000 \alpha \\
\alpha = \frac{300}{8000} = 0.0375 \text{ rad/s} \]

Use angular kinematics:

\[ \omega = \omega_0 + \alpha t \]

\[ \omega = 0 + (0.0375)(5) \]

\[ \omega = 0.19 \text{ rad/s} \]

9. At the end of the life of some stars, gravity collapses the star's core and makes a very compact remnant called a "neutron star". Assume a dying star's core (initial radius 10⁸ m) collapses into a neutron star (final radius 10⁴ m). If the stellar core was rotating at an initial angular velocity of \( \omega_i = 10^{-6} \text{ rad/s} \) before the collapse, what is the final angular velocity of the much smaller neutron star? Assume both the stellar core and the neutron star have the same mass \( M = 10^{30} \text{ kg} \), and a moment of inertia of \( I = \frac{2}{5} MR^2 \).

\[ \begin{align*}
& \text{a. } 10 \text{ rad/s} \\
& \text{b. } 10^2 \text{ rad/s} \\
& \text{c. } 10^3 \text{ rad/s} \\
& \text{d. } 10^4 \text{ rad/s} \\
& \text{e. } 10^6 \text{ rad/s}
\end{align*} \]

\[ I_i = \frac{2}{5} M (10^{16}) < R_i = 10^3 m \quad \omega_i = 10^{-6} \text{ rad/s} \]

\[ I_f = \frac{2}{5} M (10^9) < R_f = 10^4 m \quad \omega_f = ? \]

\[ M_i = M_f = 10^{30} \text{ kg} \quad \text{(irrelevant but could use if you want)} \]

Cons. of angular momentum:

\[ L_f = L_i \]

\[ I_f \omega_f = I_i \omega_i \]

\[ \frac{2}{5} M (10^9) \omega_f = \frac{2}{5} M (10^{16}) (10^{-6} \text{ rad/s}) \]

\[ 10^9 \omega_f = 10^{16} \times 10^{-6} \]

\[ \omega_f = 10^2 \text{ rad/s} \]
10. A motor-driven ferris wheel is in a state of linear and rotational equilibrium. Which statement(s) below is (are) true?

- a. There must be inhabitants of equal mass in each car of the ferris wheel.
- b. The ferris wheel must not be rotating.
- c. The wheel is experiencing no angular or tangential acceleration.
- d. Both (b) and (c) above.
- e. All of the above.

**Definition of equilibrium**

11. Consider two cylindrical objects both of height 1m and circular end area 5m². I apply different forces to these objects, as shown in the figure below. To **object A**, I apply a force of F and it squishes by 0.5 m. To **object B**, I apply a force of 2F and it compresses by 0.1 m. How does the Young’s modulus for object A compare to that of object B?

- a. Object A has a larger modulus.
- b. **Object B has a larger modulus**.
- c. They both have the same modulus.
- d. None of the above; Young’s modulus does not relate to this problem.

**F applied to top surface**

\[ \frac{F}{A} = Y \frac{\Delta L}{L_0} \]

**Obj. A**

\[ F = 1 \text{N} \]
\[ \Delta L = 0.5 \]
- weaker force, lots of compression

\[ \frac{1 \text{N}}{5 \text{m}^2} = Y \frac{0.5 \text{m}}{1 \text{m}} \]
\[ Y_A = 0.4 \]

**Obj. B**

\[ F = 2 \text{N} \]
\[ \Delta L = 0.1 \text{m} \]
- stronger force, less compression, material is less squishy so Young’s modulus must be higher.

Let’s check... \[ \frac{2 \text{N}}{5 \text{m}^2} = Y \frac{0.1 \text{m}}{1 \text{m}} \]
\[ Y_B = 4 \]
- larger!
12. Air pressure at sea level is approximately 100,000 Pa, but at the top of Mount Everest, the air thins to a meagre 30,000 Pa. The bulk modulus for glass is $5.3 \times 10^{10}$ Pa. If a glass sculpture has a volume 0.7 m$^3$ at sea level, by how much will its volume change due to the change in pressure if you bring it to the top of Everest? Assume the sculpture remains at a constant temperature. $→$ no thermal expansion/contraction

a. It will shrink by $9 \times 10^{-7}$ m$^3$

b. It will grow by $9 \times 10^{-7}$ m$^3$

c. It will shrink by $5 \times 10^{-3}$ m$^3$

d. It will grow by $5 \times 10^{-3}$ m$^3$

e. It will stay the same size.

Volume Compression due to pressure change.

$\Delta p = -B \frac{\Delta V}{V}$

$\Delta V = ?$

$30,000 - 100,000 = -(5.3 \times 10^{10}) \frac{\Delta V}{0.7 \text{ m}^3}$

$\frac{-5.3 \times 10^{10}}{-70,000} = \Delta V$

$\Delta V = 9.25 \times 10^{-7} \text{ m}^3$ positive $→$ it grows.

(can also think: less pressure at high altitude, so it should expand)

13. Ice has a density of 917 kg/m$^3$, and ocean salt water has a density of 1025 kg/m$^3$. You see the point of an iceberg sticking out of the water. Rounded to the nearest percent, what percentage of the whole iceberg is beneath the surface of the water?

a. 18%

b. 62%

c. 89%

d. 94%

e. 112%

$\rho_{\text{ice}} = 917 \text{ kg/m}^3$ $\quad \rho_{\text{water}} = 1025 \text{ kg/m}^3$

$V_{\text{above}} = 5 \text{ m}^3$

Use Archimedes' principle.

$\frac{V_{\text{fluid-displaced}}}{V_{\text{object}}} = \frac{\text{fraction under water}}{\text{fluid}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$

$= \frac{917}{1025} = 0.8946$

$= 89\%$
14. The maximum depth of Cheat Lake is about 27 meters. What is the pressure of the water in Cheat Lake at that depth? Note that Cheat is a freshwater lake, and a typical air pressure for Morgantown is 102133 Pascals. - air pressure matches water surface pressure

\[ \rho_0 = 102133 \text{ Pa} \]
\[ h = 27 \text{ m} \]
\[ \rho = 10^3 \text{ kg/m}^3 \]
\[ g = 9.8 \text{ m/s}^2 \]

\[ \rho = \rho_0 + \rho g h \]
\[ \rho = 102133 + (10^3)(9.8)(27) \]
\[ \rho = 102133 + 264600 \]
\[ \rho = 366733 \text{ Pa} \approx 3.7 \times 10^5 \text{ Pa} \]

15. I have to use 150 N of force to hold an inflated ball at rest just below the surface of Cheat Lake. What is the volume of the ball (assuming it does not compress at all due to water pressure)?

- Buoyancy force pushes upwards.

\[ F_{\text{push}} = 150 \text{ N} \rightarrow \text{balanced by buoyancy force, so } F_B = F_{\text{push}} = 150 \text{ N} \]

\[ F_B = \rho \frac{V g}{\text{fluid}} \rightarrow \text{object.} \]

\[ 150 \text{ N} = (10^3)(V)(9.8) \]

\[ V = \frac{150}{1800} = 0.0153 \text{ m}^3 \]
16. Which object has a larger moment of inertia: a donut or a round cake? Assume for the sake of this problem that they both have the same mass and outer radius.

a. The donut.
b. The cake.
c. They both would have the same moment of inertia.
d. The answer depends on the density of their batter.

\[ I = \Sigma m r^2 \]

\[ \text{Donut} \quad \text{Cake} \]

more mass at larger radii \(\Rightarrow\) larger \(I\).

mass in center \(\Rightarrow\) smaller \(I\).

17. A wide pipe in my house has a cross-sectional area of 0.008 m². Water flows through that pipe and attaches to my garden hose, which has a spout at the end with a cross-sectional area of 0.001 m². If water is flowing through the wide pipe at a velocity of 1.5 m/s, at what velocity will the water exit the end of my garden hose?

a. 8 m/s
b. 12 m/s

c. 15 m/s

d. 24 m/s

e. It depends on how high above the ground I hold the hose.

\[ A_p = 0.008 \text{ m}^2 \quad A_h = 0.001 \text{ m}^2 \]

\[ V_p = 1.5 \text{ m/s} \]

\[ A_p V_p = A_h V_h \]

\[ 0.008 \times 1.5 = 0.001 \times V_h \]

\[ V_h = \frac{0.008 \times 1.5}{0.001} = 12 \text{ m/s} \]
18. A cube of mass 10 kg and volume 2 m$^3$ is sitting on the floor. What pressure that the cube's bottom surface has on the floor? Note that the volume of a cube is its length cubed.

\begin{align*}
\text{V} &= 2 \text{ m}^3 = \ell^3 \\
\text{So area touching floor} &= \ell^2 \\
\ell &= \sqrt[3]{2} = 1.26 \text{ m} \\
A &= (1.26 \text{ m})^2 = 1.59 \text{ m}^2
\end{align*}

\[ p = \frac{F}{A} \]

\[ p = \frac{F_g}{A} = \frac{mg}{A} = \frac{(10 \text{ kg})(9.8 \text{ m/s}^2)}{1.59 \text{ m}^2} \]

\[ p = 61.6 \text{ Pa} \]
19. I look out the window and see that my scientific thermometer reads 252 K. Brrr! About how many degrees Celsius is this below the boiling point of water (100°C)?

\[ T_c = T_k - 273.15 \]

\[ T_c = 252 - 273.15 = 21.15 \text{°C} \]

This is below boiling point.

20. The linear coefficient of thermal expansion for concrete is \(12 \times 10^{-6}/\text{°C}\). You are building a walking path on your yard with concrete slabs, each 2m in length. You know about thermal expansion and want to leave space between each slab so they don’t buckle. What is the minimum space should you leave between adjacent slabs, assuming a typical Winter/Summer range in the temperatures of your local climate of -10 °C to 30 °C?

\[ \alpha = 12 \times 10^{-6}/\text{°C} \]

\[ \Delta L = \alpha L_0 \Delta T \]

\[ \Delta T = -10 + 30 = 40 \text{°C} \]

\[ \Delta L = 12 \times 10^{-6} \times 2 \times 40 = 0.00096 \text{m} = 0.096 \text{cm} \]