Recap: Vectors

- Vector components

\[ \vec{A} \]

\[ |\vec{A}|^2 = A_x^2 + A_y^2 \]

- Vector translations

If you were to add the red and blue vectors, roughly what direction would your result point?

A. 

B. 

C. 

D. 

E. None of the above

Your intuitive understanding of the Physical world

- Tested basic perception of Physics principles.
- We're going to work on fixing your misconceptions!
- For instance, horizontal and vertical motion act independently!
Today!

- Decomposing X and Y movements.
- How to hit a target.
- How to go about Projectile Motion problems.

Ignoring air resistance, what would be the path of motion if someone ran off of a cliff?

Three velocity vectors.

\[ v = v_x \]

Starts with all motion in horizontal velocity, \( v_x \)

(no vertical velocity, \( v_y = 0 \) m/s)
Free fall!

$t = 0s$

$v = v_{x0}$

$a_x = -9.8 \text{ m/s}^2$

Throughout the fall, what is the jumper’s horizontal acceleration, $a_x$?

A. $v_{x0}^2$
B. $+9.8 \text{ m/s}^2$
C. 0 m/s$^2$
D. $-9.8 \text{ m/s}^2$
E. Not given, so unknown

$v_x = v_{x0} + at$

- In free bodies, there is no horizontal acceleration
- $v_x$ is always equal to $v_{x0}$ (until the projectile hits something)

Free fall!

$[\text{Horizontal motion}]$

$t = 0s$

$v_x = v_{x0}$

$t = 1s$

$v_x = v_{x0}$

$t = 2s$

$v_x = v_{x0}$

$t = 3s$

$v_x = v_{x0}$
Free fall!  
[Vertical motion]

1. \( t = 0 \) \( v_x = v_{xo} \)
2. \( t = 1x \) \( v_x = v_{xo} \) \( a_y = -9.8 \text{ m/s}^2 \)
3. \( t = 2x \) \( v_x = v_{xo} \)
4. \( t = 3x \) \( v_x = v_{xo} \)

\[ v_y = v_{yo} \]

\[ v = v_x + at \]

\[ \Delta x = v_x t + \frac{1}{2} at^2 \]

\[ v^2 = v_{xo}^2 + 2a\Delta x \]

How far downward has the jumper fallen after 3 seconds?

Free fall!  
[Net velocity]

\[ v = v_x + at \]

\[ v^2 = v_{xo}^2 + v_{yo}^2 \]

Generic Projectile Motion

y motion: Same as vertical-only problem!  
x motion: Covers \( \Delta x \) distance at \( v_{xo} \) speed
$x$ and $y$:
Motions are Independent
- Separation of vectors into components allows separations of equations into components:
  \[
  v_x = v_{xa} + a_x t \\
  v_y = v_{ya} + a_y t \\
  \Delta x = v_{xa} t + \frac{1}{2} a_x t^2 \\
  \Delta y = v_{ya} t + \frac{1}{2} a_y t^2
  \]
For projectiles:
  \[
  a_x = 0 \text{ m/s}^2 \quad a_y = \pm g = \pm 9.8 \text{ m/s}^2
  \]

Break up what you know in terms of the horizontal and vertical...
**Prevents mistakes!**

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
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“After it leaves your hand, before it hits the ground”
- $v_0$ = initial velocity vector
- $\theta_i$ = initial direction of velocity vector
- $v_x = 0$ at top of trajectory
- $v_y = v_{ya}$ remains the same throughout trajectory because there is no acceleration along the x-direction

**Strategy for Projectile Motion Problems**
Beyond this, projectile motion problems just take a lot of planning and thinking.
Take your time and think about the set-up of the problem.

  - What do I know?
  - What’s the first step?
  - What’s the next step?
Strategy for Projectile Motion Problems

The time will be the same for \( x \) and \( y \) parts of the question.

If you don’t have enough information for \( x \) or \( y \) components, solve for time.

Tossing something from a roof
(5 examples with increasing difficulty)

A ball is launched from the edge of a 15.0m tall building at 16 m/s at an angle of 60 degrees from the horizontal.

1. How much time does it take to fall?
2. How far from the base of the cliff does it hit the ground? (Need the time first)
3. How fast it is moving vertically when it hits the ground?
   (y component of final velocity)
4. What is the magnitude of its velocity when it hits the ground?
5. What is the angle that it hits the ground from the horizontal?

How much time to fall?
(Think Vertical)

A ball is launched from the edge of a 15.0m tall building at 16 m/s at an angle of 60 degrees from the horizontal.

We’re talking about something falling, and that is vertical motion, so we will only use vertical ideas and numbers.

\[ v = v_0 + at \]
\[ \Delta x = v_0 t + \frac{1}{2} at^2 \]
\[ v^2 = v_0^2 + 2a\Delta x \]

How far from cliff base?
(Think Horizontal)

A ball is launched from the edge of a 15.0m tall building at 16 m/s at an angle of 60 degrees from the horizontal.

We know it was in the air for 3.66s (from the previous question), and it’s moving at a constant speed in the \( x \)-direction the whole time \( (a_x = 0) \).

\[ v = v_0 + at \]
\[ \Delta x = v_0 t + \frac{1}{2} at^2 \]
\[ v^2 = v_0^2 + 2a\Delta x \]
**Vertical speed when it lands?**
*(Think Vertical)*
A ball is launched from the edge of a 15.0m tall building at 16 m/s at an angle of 60 degrees from the horizontal.

**Asking for y component of final velocity.**
It’s been accelerating down the whole time. We know that gravity is causing this acceleration, so we can figure out how fast it is going (vertically) when it hits the ground.

\[
v = v_y + at
\]
\[
\Delta x = v_y t + \frac{1}{2} at^2
\]
\[
v^2 = v_y^2 + 2a\Delta x
\]

**Magnitude and Angle that it hits the ground?**
*(Finally combine!)*
A ball is launched from the edge of a 15.0m tall building at 16 m/s at an angle of 60 degrees from the horizontal.

Final angle: which way is the ball going?

How do we get is its final velocity vector?

\[
v = v_y + at
\]
\[
\Delta x = v_y t + \frac{1}{2} at^2
\]
\[
v^2 = v_y^2 + 2a\Delta x
\]