Radiative Line Transfer and Detailed Balance

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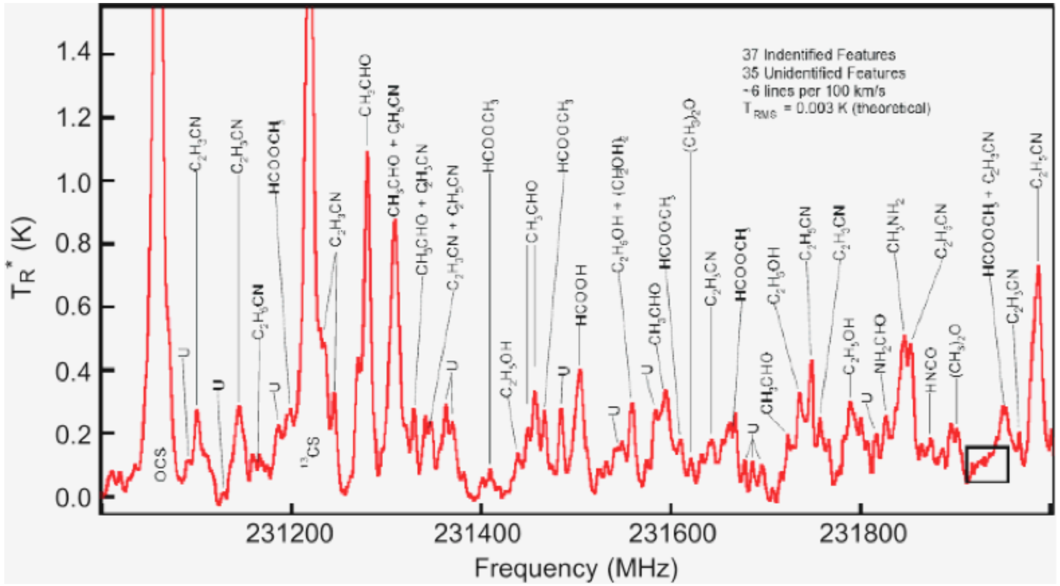
Concept Map

- Line Radiative Transfer
  - Einstein Coefficients
  - Radiative Transfer and Detailed Balance
- Masers
  - Applicability to observational astronomy

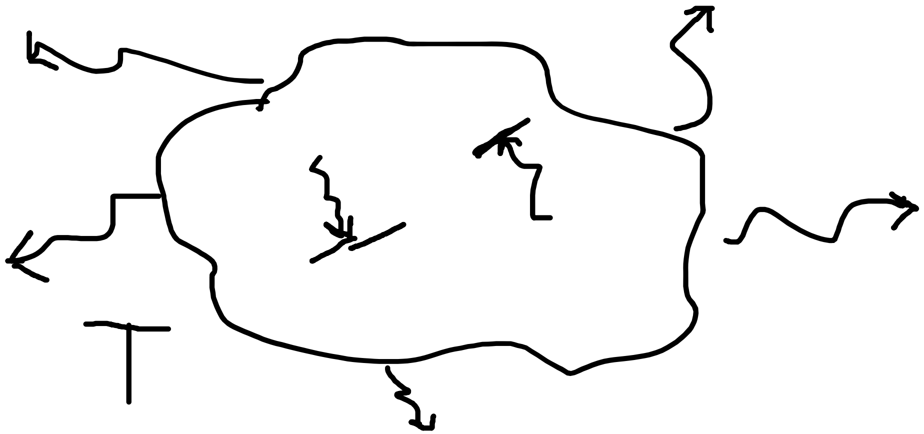


CAUTION!

There are a lot of abstract concepts in this section of ERA and we don’t have enough time to go into minute detail. I highly recommend reading through the material on your own (as you should for every topic).

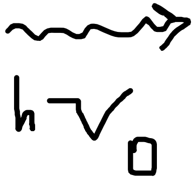
Line Radiative Transfer: Einstein Coefficients

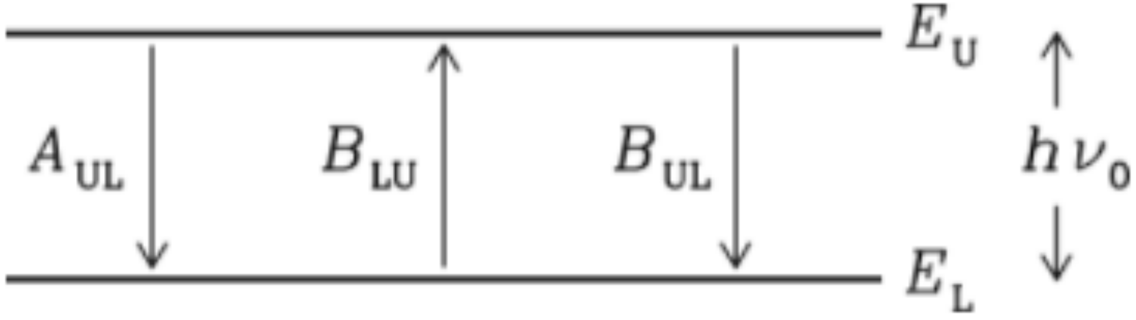
- 3 types of emission:
  - Spontaneous emission (A_{UL})
  - Absorption (B_{LU})
  - Stimulated emission (B_{UL})
    - **Negative absorption**



- For our model (two-level system):

$$E = E_U - E_L$$





- Which leads to a photon: $\nu_0 = \frac{E}{h}$

Line Radiative Transfer: Einstein Coefficients

- As outlined in the recombination lines lecture, there is an intrinsic line width to spectral lines.
- For absorption, the coefficient is dependent on the incident radiation field.
- We define the **profile-weighted mean radiation energy density**:

$$\bar{u} \equiv \int_0^\infty u_\nu(\nu)\,\phi(\nu)\,\mathrm{d}\nu \longrightarrow$$

$$B_{LU}\,\bar{u}$$

&

$$B_{UL}\,\bar{u}$$

Line Radiative Transfer: Einstein Coefficients

- In thermodynamic equilibrium (TE) we have stationary states.
- Average rate of emission of photons must balance the average rate of absorption of photons from the radiation field.

$$n_U \, A_{UL} + n_U \, B_{UL} \, \bar{u} = n_L \, B_{LU} \, \bar{u}$$

- Solving for ū connects the properties of the quantum system to the blackbody radiation field:

$$\bar{u} = \frac{A_{UL}}{(n_L/n_U) \, B_{LU} - B_{UL}}$$

Line Radiative Transfer: Einstein Coefficients

- The Boltzmann equation gives

$$\frac{n_U}{n_L} = \frac{g_U}{g_L}\, exp\left(-\frac{h\,\nu_0}{k\,T}\right)$$

where g_U and g_L are called statistical weights.

- Combining this with ū from the previous slide:

$$\bar{u} = A_{UL}\left[\frac{g_L}{g_U}\, exp\left(\frac{h\,\nu_0}{k\,T}\right)\, B_{LU} - B_{UL}\right]^{-1}$$

Line Radiative Transfer: Einstein Coefficients

- When we use the Planck radiation law (2.86) for $B_\nu(T)$ near $\nu = \nu_0$:

$$\bar{u} \approx \frac{4\pi}{c} \frac{2h\nu_0^3}{c^2} \left[exp\left(\frac{h\,\nu_0}{k\,T} \right) - 1 \right]^{-1}$$

- This must agree with our equation for $\bar{u}$ from the previous slide for *all* temperatures *T*.
- Which leads to…

Line Radiative Transfer: Equations of Detailed Balance

- The equations of detailed balance:

$$\frac{g_L}{g_U}\frac{B_{LU}}{B_{UL}} = 1 \qquad \frac{A_{UL}}{B_{UL}} = \frac{8\pi h\nu_0^3}{c^3}$$

- Some takeaways:
  - If we know one of the three coefficients, we can determine the other two.
  - B_{LU} cannot be zero; stimulated emission **must** occur.

Radiative Transfer

- We go back to Chapter 2 for the equation of radiative transfer:

$$\frac{dI_\nu}{ds} = -\kappa\, I_\nu + j_\nu$$

- For pure absorption:

$$\frac{dI_\nu}{ds} = -\left(\frac{h\nu_0}{c}\right) n_L B_{LU} \phi(\nu) I_\nu$$

- For stimulated emission:

$$\frac{dI_\nu}{ds} = \left(\frac{h\nu_0}{c}\right) n_U B_{UL} \phi(\nu) I_\nu$$

Radiative Transfer

- Adding these two yields the **net absorption coefficient**:

$$\kappa = \left(\frac{h\nu_0}{c}\right)\left(n_L B_{LU} - n_U B_{UL}\right)\phi(\nu)$$

- For spontaneous emission:

$$\frac{dI_\nu}{ds} = j_\nu = \left(\frac{h\nu_0}{4\pi}\right) n_U A_{UL} \phi(\nu)$$

- Now we can write the full spectral-line equation of radiative transfer:

$$\frac{dI_\nu}{ds} = -\left(\frac{h\nu_0}{c}\right)\left(n_L B_{LU} - n_U B_{UL}\right)\phi(\nu) I_\nu + \left(\frac{h\nu_0}{4\pi}\right) n_U A_{UL} \phi(\nu) I_\nu$$

Radiative Transfer

- Interestingly, we can use these equations to eliminate A_{UL} , B_{UL} , and B_{LU} and Kirchhoff's law in LTE to recover the Boltzmann equation.
- Our derivations are not specific to total TE, but also LTE!
- Using this, the **net opacity coefficient** in LTE is:

$$\kappa(\nu) = \frac{c^2}{8\pi\nu_0}\frac{g_U}{g_L}\, n_L\, A_{UL}\left[1 - exp\left(-\frac{h\nu_0}{kT}\right)\right]\,\phi(\nu)$$

Radiative Transfer

- Remember, at radio frequencies $h\nu/kT << 1$ leading to stimulated emission nearly cancelling pure absorption and significantly reducing line opacity.
- Also, as $\kappa \alpha 1/T$ and $B_\nu \alpha T$ the product κB_ν is independent of line temperature.
- **The brightness of an optically thin radio emission line is proportional to the column density of emitting gas but can be nearly independent of the gas temperature**.
  - Question: If we observe the HI line flux of an optically thin galaxy, what can we interpret from it?

Excitation Temperature

- When our two-level system is not in LTE, its excitation temperature is defined by:

$$\frac{n_U}{n_L} \equiv \frac{g_U}{g_L} exp\left(-\frac{h\nu_0}{kT_x}\right)$$

- *T_x* causes collisional excitations and de-excitations.
- We need to revise our detailed balance equation:

$$n_U(A_{UL} + B_{UL}\bar{u} + C_{UL}) = n_L(B_{LU}\bar{u} + C_{LU})$$

Masers

- If the upper energy level is overpopulated ($n_U / n_L > g_U / g_L$) then T_x is negative and the net line opacity is negative
- Huh???
- The source is actually brighter due to the medium.
- This is called maser (microwave amplification by stimulated emission of radiation) amplification and it is very common at radio frequencies.
- Can have line brightness temperatures as high as 10^{15} K!

Masers

- Assume $g_U = g_L$.
- We will also assume the line profile is a Gaussian with FWHM $\Delta\nu$ and (from last lecture) we can use the numerical approximation $\phi(\nu_0) \approx 1/\Delta\nu$.
- Then:

$$\tau = \frac{h\nu_0 B}{c\Delta\nu} \int (n_U - n_L) \mathrm{d}s$$

is called the **maser gain** and amplifies the signal by a factor exp($|\tau|$).

- s > 10^{13} cm (about 10^{-5} pc, 1 AU) for significant gain to occur.

Masers

- Masers are like lasers – they need to be “pumped” by an energy source or the upper energy levels are depopulated quickly.
- The maser is **saturated** if the stimulated emission rate is limited by the pump luminosity; it is **unsaturated** if the pump is more than adequate.

- Where is a good place to look for masers?

Masers

- Supermassive black holes!
- Masers are great sources for measuring astrometry (motions) and thus are a great tool for measuring black hole mass.

$$V_z = \sqrt{\frac{GM}{R}}\sin\theta$$

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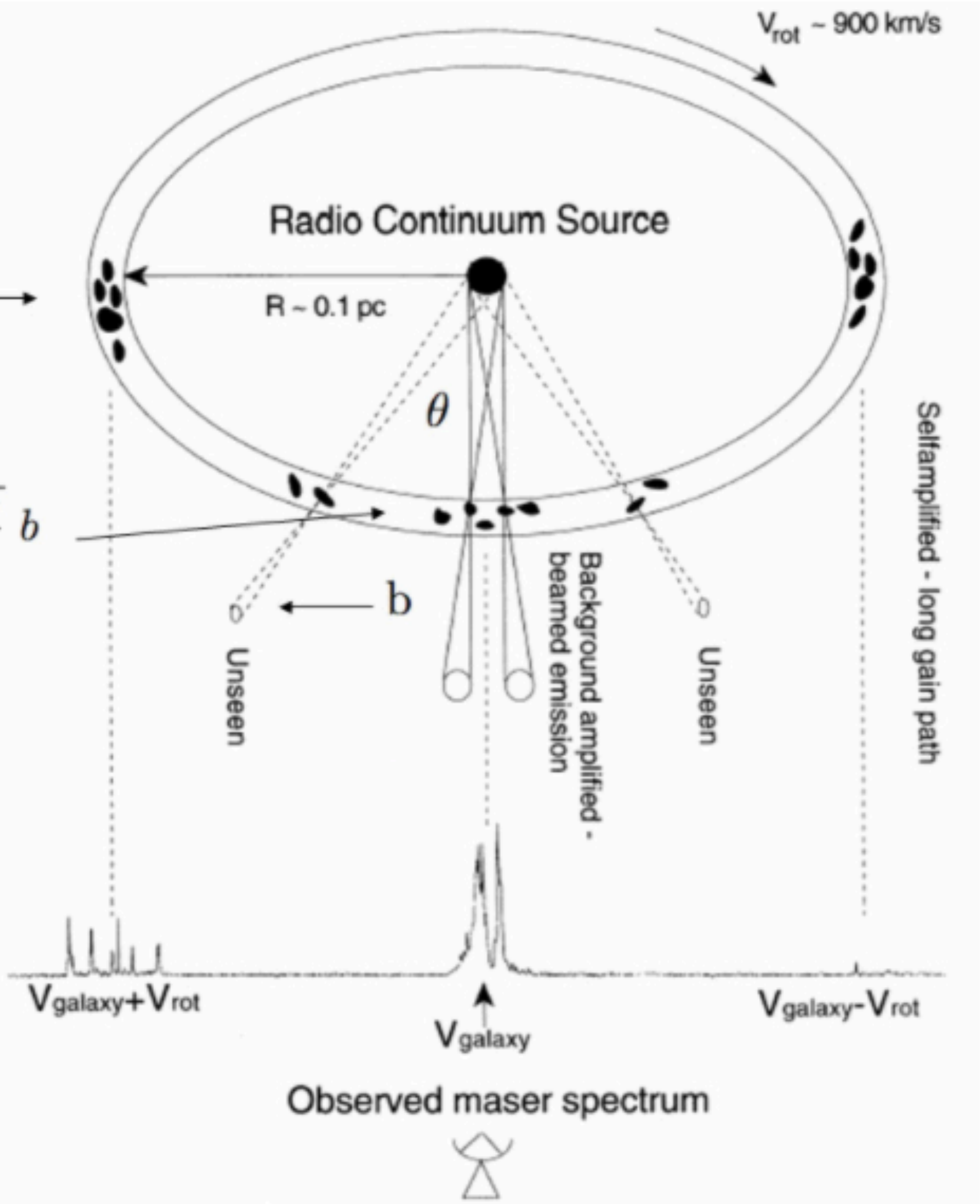
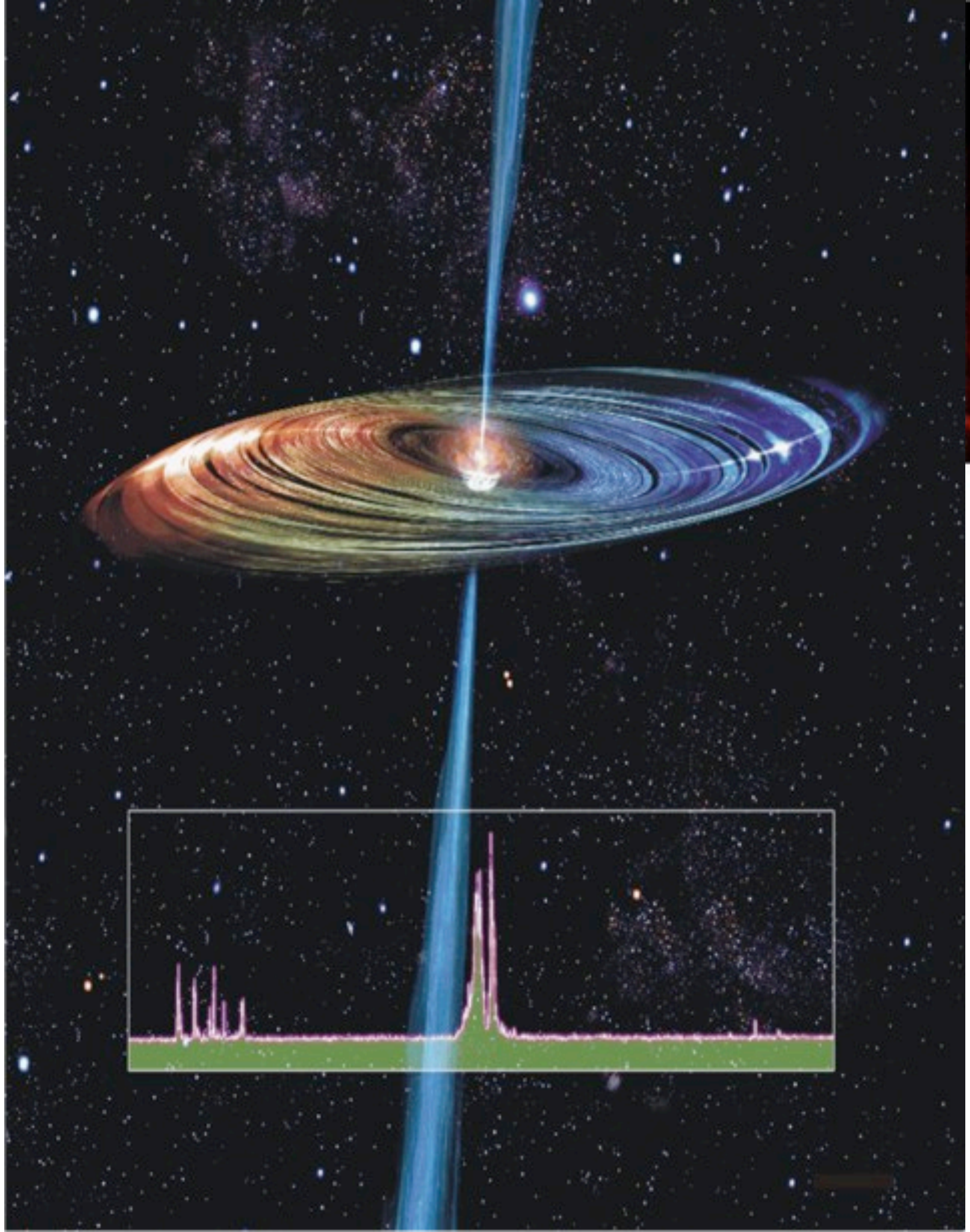
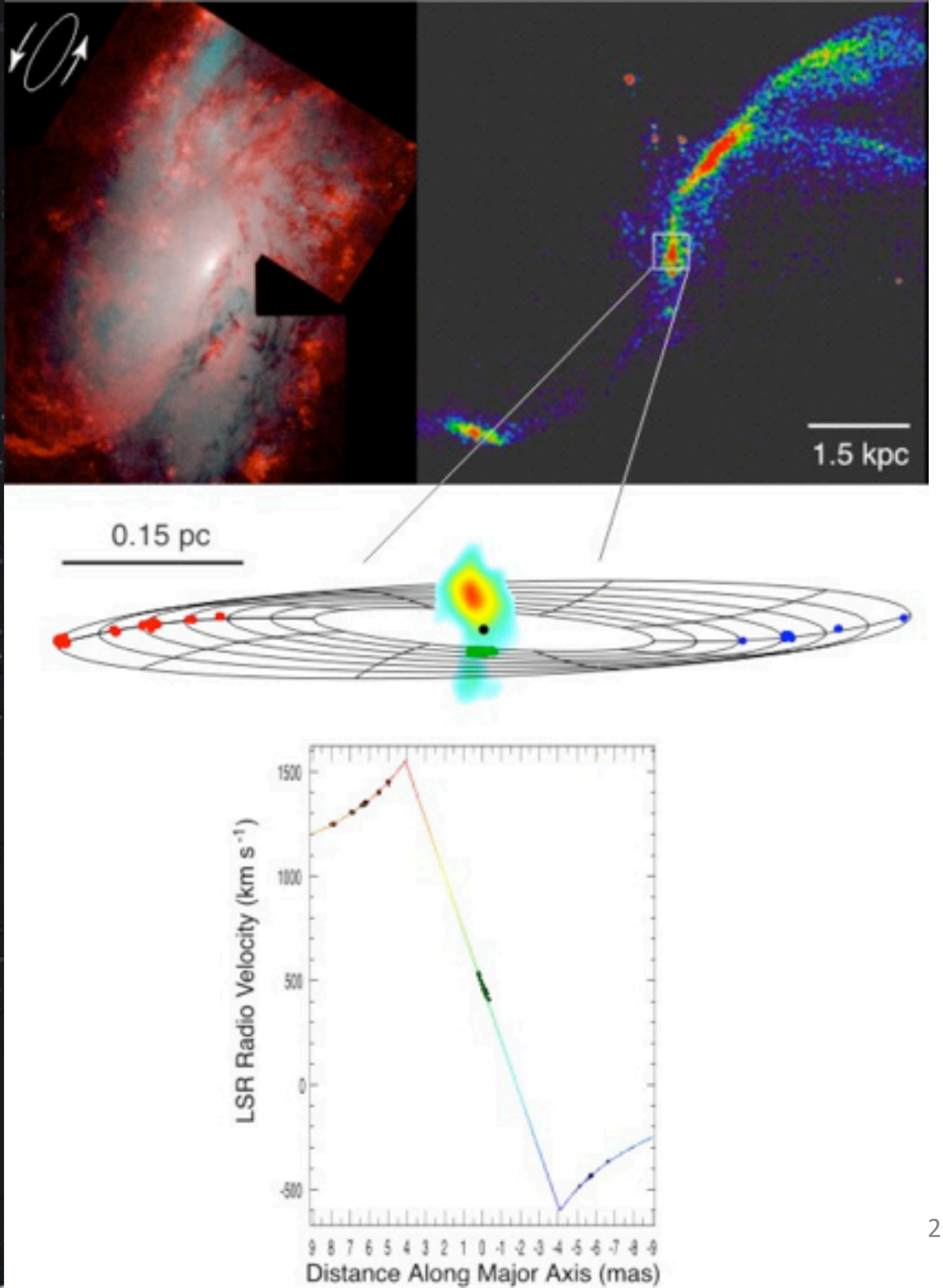
$$V_z = \sqrt{\frac{GM}{R}}\,\frac{b}{R} = \sqrt{\frac{GM}{R^3}}\ b$$





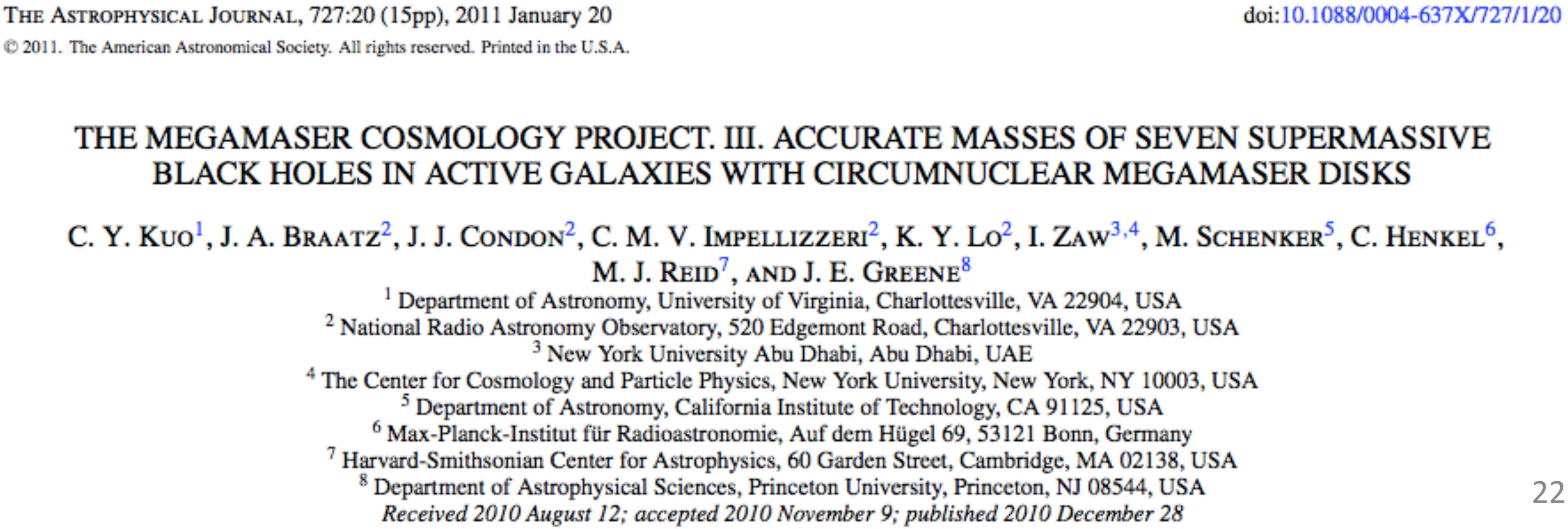

1.5 kpc
0.15 pc
LSR Radio Velocity (km s⁻¹)
1500
1000
500
0
-500
9 8 7 6 5 4 3 2 1 0 -1 -2 -3 -4 -5 -6 -7 -8 -9
Distance Along Major Axis (mas)

Masers: Question for you

- Using the information from the figure, what is the mass of the object causing the Keplerian orbits of these masers? Find the mass density of this object. Compare it to a star cluster of mass 1000 solar masses and radius 10 pc.
  - V_{rot} = 900 km/s
  - V_{galaxy} = 450 km/s
  - R = 0.1 pc
  - V =$\sqrt{GM/R}$

Masers

- There is an ongoing project called the Megamaser Cosmology Project which uses masers to measure all sorts of extragalactic parameters for many systems.



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THE MEGAMASER COSMOLOGY PROJECT. III. ACCURATE MASSES OF SEVEN SUPERMASSIVE BLACK HOLES IN ACTIVE GALAXIES WITH CIRCUMNUCLEAR MEGAMASER DISKS

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