ASTR469 Lecture 2: Quantifying Light (Ch. 5)

Assess yourself/study guide after lecture & reading (without peeking at notes)...

1. What measure would you use to determine the energy output of a source, and what are its units?

2. What are the units of the energy that hits a detector (flux)? [Note: your detector has a finite size.]

3. You're looking at an object with constant intensity as a function of angular position. It appears round, with an angular radius of 4". You measure its spectral intensity as 10^4W/Hz/m^2/sr. What is the total flux detected from the whole object?

4. You observe the flux of a point-source object in the distance to be 5 $\mathrm{W\,m^{-2}\,Hz^{-1}}$. If it is 5 km away, what is the luminosity of the source?

5. Let's say you have a detector operating at 1 THz frequency. The detector has a collecting area of $2\,\mathrm{m}^2$ and is pointed directly at the target. How many photons per second at that frequency are you detecting from an object of $F_{1THz} = 10^{20}\,\mathrm{W\,m^{-2}\,Hz}$?

1 Light from astronomical objects

Most astronomical objects emit light that changes as you tune in to different frequencies/wavelengths/energies in the electromagnetic spectrum. Next time we'll talk about what kind of emissions happen, but today we're going to discuss how astronomers quantify the light that is received on Earth within a particular frequency range (light within frequency interval $\nu + d\nu$).

We can separate observations into *intrinsic* properties of the astrophysical sources and *measured* properties that we observe. The difference is that measured properties depend on your distance from the source, whereas intrinsic properties do not. But before we start with the idea of receiving light on Earth, let's think about the actual emission of light from an object.

2 Intrinsic source properties: Energy generated; Luminosity

Observable astronomical sources emit light due to various mechanisms (blackbody, synchrotron, free-free emission, and other mechanisms we will later discuss). Those processes produce electromagnetic waves, and those waves carry energy away from the source. As we saw last class, a single photon has an energy that depends on its frequency: $E = h\nu$.

Luminosity is a fundamental value that tells you how much energy per second is leaving the surface an object. If luminosity is measured at a given frequency, this can also tell you how many photons the source has emitted within a certain time frame!

Consider a 100 W light bulb; it emits 100 J/s of energy. This is its *luminosity*, also sometimes referred to as *power*. We typically use units of erg/s or W (J/s). I will mostly stick with the latter because it represents S.I. units. The Solar luminosity is 3.828×10^{26} W, so basically the Sun's intrinsic energy output is the equivalent of about 4×10^{24} light bulbs.

OK, but what if we wanted to know how much luminosity was coming out at a given frequency? In this case we use the "spectral luminosity," L_ν, which tells us the luminosity that would be observed if you could only see at a particular frequency. The units of L_ν are W/Hz, because it is the luminosity per frequency interval $d\nu$.

The "bolometric luminosity" L, which we were previously discussing, is simply the integral of the spectral luminosity over all frequencies:

$$L = \int_0^\infty L_\nu d\nu \ . \tag{1}$$

What we usually measure in astronomy is the luminosity in a particular frequency band. But, if we measure it at a number of frequency bands, we can fit a spectrum that has a known form and physical origin (like a blackbody spectrum, for instance, which we will discuss later). Then, we can integrate that to get the total luminosity.

Side note and sneak-preview... you have probably encountered luminosity before with the Stephan-Boltzmann law:

$$L = A\sigma T^4 \,, \tag{2}$$

where A is the emitting area, σ is the Stephan-Boltzmann constant, and T is the surface temperature. This equation gives the total luminosity emitted from a blackbody-emitting object. For spherical objects, of course $A = 4\pi R^2$ because the light is going in all directions at the same time. This formula is applicable for stars.

Let's move on now to measured properties.

Intensity

Intensity is a measured quantity, but if we consider light travelling through a vacuum, its intensity will remain the same. While I (and many others) find this annoying and confusing, I wanted to preface this by saying that it's mostly confusing because we equate the word "intensity" with "how bright something appears to be". However, our perception has more to do with **flux**, so let's leave perception out of this. Note that a source's intensity may be lost if there is absorption or some other attenuating effect between you and the source.

The *spectral intensity* of radiation, I_ν, is the most basic observable quantity. Here, *spectral* refers to the fact that it is measured at one particular wavelength (thus the ν subscript). The spectral intensity has units of energy, per unit time, per unit area, per unit frequency, per unit *steradian*, as discussed last class. The spectral intensity therefore has units of J s^{-1} cm^{-2} Hz^{-1} sr^{-1} (or in wavelength form, J s^{-1} cm^{-2} m^{-1} sr^{-1} or fill in whatever wavelength units you want to use: m, nm, Angstrom).

The spectral intensity goes by many other names, by the way, which can make things even more confusing; sometimes people call it specific intensity, radiance, irradiance, brightness, or surface brightness. Annoyingly, sometimes people also say "brightness" and mean "flux". If someone is being ambiguous, always ask them what the units are of whatever they are talking about.

Two important things about the spectral intensity:
(1) It is independent of distance, if the light travels through free space. Thus, the camera exposure time and aperture setting for an exposure of the Sun would be the same, regardless of whether the photograph was taken close to the Sun (from near Venus, for example) or far away from the Sun (from near Mars, for example), so long *as the Sun is resolved* in the photograph. This seems terribly wrong at first, but can easily be proven.
(2) Somewhat related to the previous point, it is the same at the source and at the detector. Thus you can think of brightness in terms of energy flowing out of the source or as energy flowing into the detector. Those two quantities will be the same.

We can conceptualize intensity as the energy dE passing through an infinitesimally small area dA by:

$$dE = I_\nu \, dA \, \cos\theta \; d\Omega \, d\nu \, dt \,. \tag{3}$$

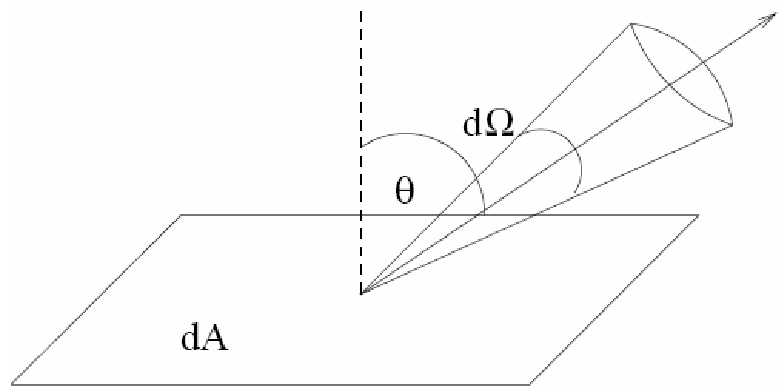

Figure 1: The geometry for spectral intensity.

Here, "spectral" refers to the fact that it is at a particular frequency. We can of course rewrite this as:

$$I_\nu = \frac{dE}{dA\,\cos\theta\,d\Omega\,d\nu\,dt}\,. \tag{4}$$

θ is measured normal to the surface dA and $d\Omega$ is the solid angle.

Notice that we wrote the spectral intensity in frequency units. I_ν has a dependence on $d\nu$, and $d\nu \neq d\lambda$. Instead, $c = \lambda\nu$ so

$$d\nu = -(c/\lambda^2)d\lambda \tag{5}$$

so combining with the above equations

$$\nu I_\nu = \lambda I_\lambda \tag{6}$$

To get the *intensity* or *integrated intensity* we would of course integrate over frequency or wavelength:

$$I = \int_0^\infty I_\nu d\nu = \int_0^\infty I_\lambda d\lambda \tag{7}$$

Flux

While intensity is perfect for extended sources, we are frequently more interested in the quantity of *flux* integrated over solid angle:

$$F_\nu = \int I_\nu \cos\theta d\Omega \tag{8}$$

or

$$F_\nu = \int_0^{2\pi}\int_0^{\pi} I_\nu \cos\theta \sin\theta d\theta d\phi\,. \tag{9}$$

The units of flux are therefore $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$ or $\mathrm{W\,m^{-2}\,Hz^{-1}}$. The integration is carried out over the solid angle of the observations, i.e. the field of view of the telescope or perhaps only one pixel.

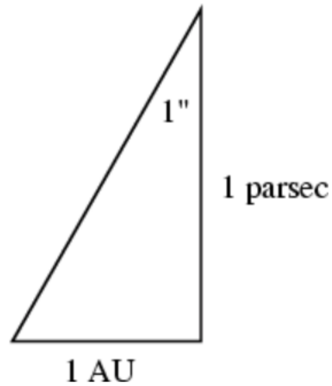

Figure 2: The definition of a parsec. Note that this implies $\tan(1") = \frac{1\text{AU}}{1\,\text{pc}}$, which is a convenient thing to remember if you forget the conversion from parsecs to meters.

Why do we care about flux when we have a perfectly good unit of spectral intensity? Remember that spectral intensity is a surface brightness. The source may not be *resolved*, that is, it may effectively be a single point of light. This is true for stars, for example, which are *unresolved* with nearly all telescopes. So, for small, unresolved *point sources*, the flux is a much better measure. For large, resolved sources the spectral intensity is prequently of interest.

We can relate the flux and luminosity with

$$L_\nu = 4\pi d^2 F_\nu \,, \tag{10}$$

where d is the distance to the source. This is another way of restating that brightness is the same measured and emitting.

3 Astronomical Distances and Source Sizes

One lingering thing: we define one "Astronomical Unit", 1 AU, as the distance between Earth and the Sun. The definition is fixed at $1\,\text{AU} = 1.496e + 11\,\text{m}$.

However, many astronomical distances are measured in "parsecs":

$$1\,\text{pc} = 3.08 \times 10^{16}\,\text{m} \tag{11}$$

The parsec is defined as the distance from us at which we will see something that is 1 AU across as subtending an angle of 1"; see Figure 2.