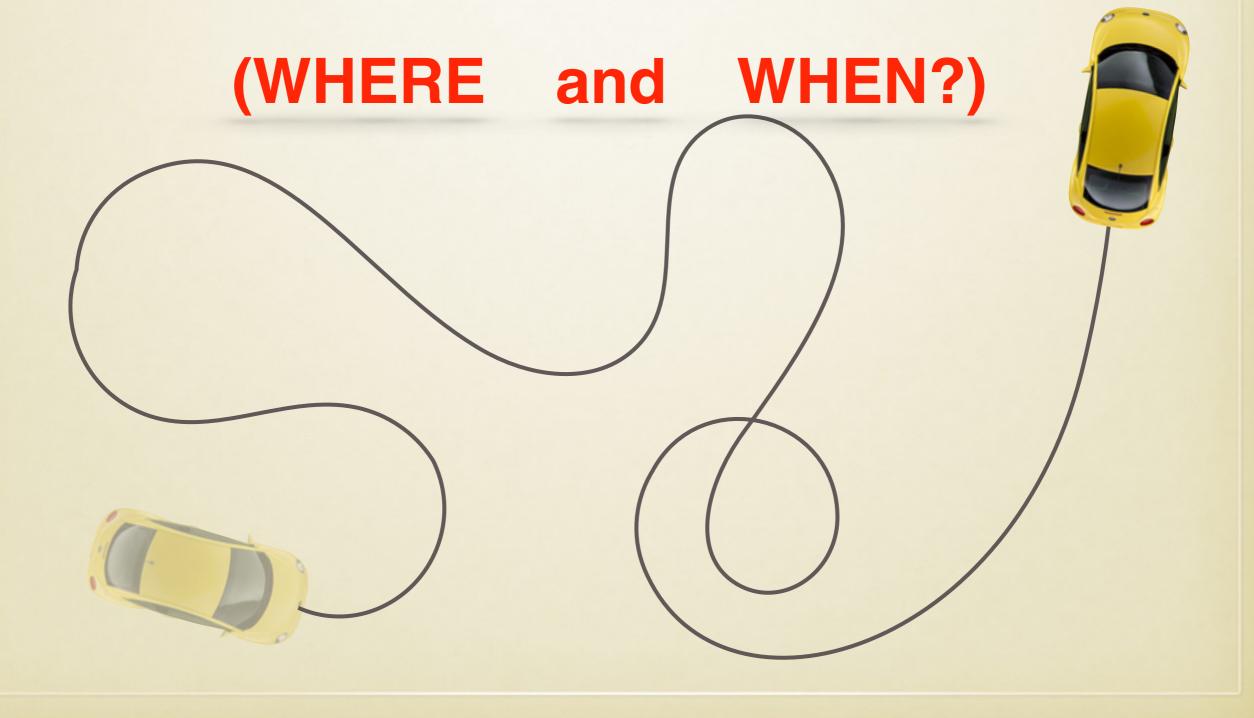
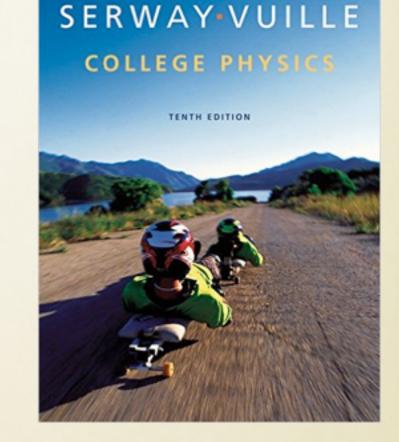
#### **Displacement, Velocity, and Acceleration**



#### Math resources

- Appendix A in your book!
  - Symbols and meaning
  - Algebra
  - Geometry (volumes, etc.)
  - Trigonometry
  - Logarithms



**Appendix A** 

#### Reminder



#### You will do well in this class by PRACTICING!

# Extra Practice Problems: 2.1, 2.3, 2.5, 2.21, 2.25, 2.27

Also: (Ungraded) homework warm-up problems

#### Reminders

Next class is next Wednesday.

Problem solving day: practicing for exam.

First clicker grade counted; BRING YOUR CLICKERS!

# **Problem Solving Pro-tips**

- 1. Draw a picture!
- 2. Use and label your reference frame.
- 3. List what you KNOW and DON'T KNOW in variable form.
- 4. Practice helps you pick best formulas!

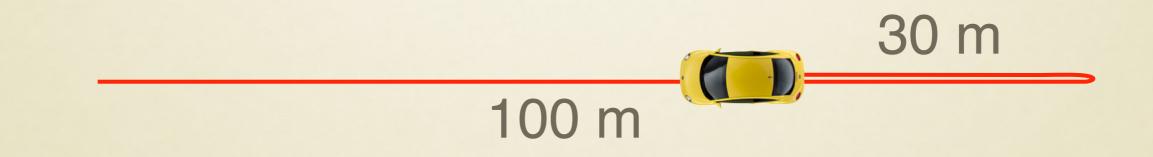
- Scalar: just a number (magnitude).
- Vector: a number (magnitude) with a direction.



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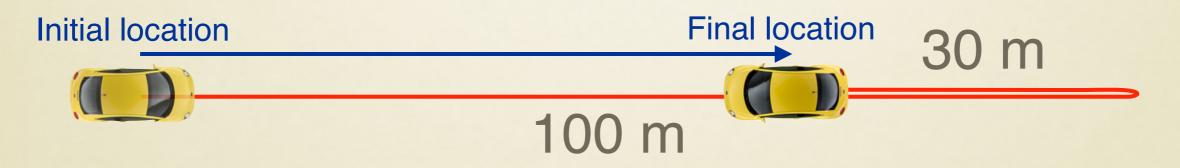
- Scalar: just a number (magnitude).
- Vector: a number (magnitude) with a direction.



Distance (scalar): 100m + 30m = 130 meters

- Scalar: just a number (magnitude).
- Vector: a number (magnitude) with a direction.

Displacement, **x** (vector): 100 - 30 = +70 meters



Distance (scalar): 100m + 30m = 130 meters

Scalars:

Vectors:

Distance, x Speed, v Displacement, **x** Velocity, **v** Acceleration, **a** 

Vectors are usually represented as BOLD (or with an arrow hat).

## **Frames of reference**





Ground's reference Driver's reference frame Velocity, v frame

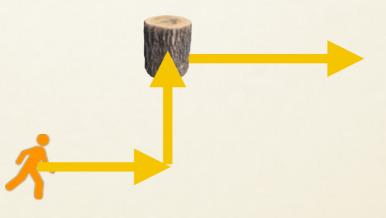
- In ground frame of reference, one car has v = +80 km/h while the other has v = +70 km/h
- In reference frame of driver, velocity of other car is
   v = +10 km/h

• PT #1: Draw a picture!

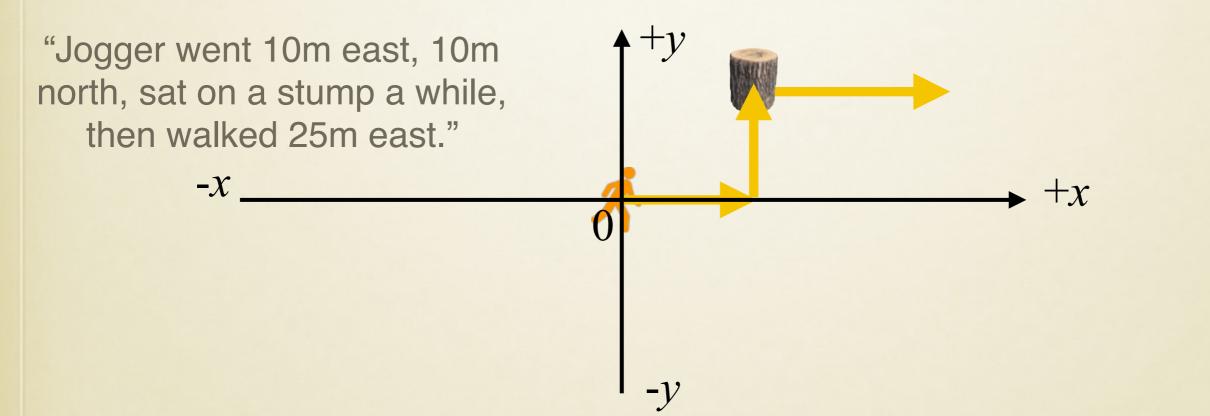
"Jogger went 10m east, 10m north, sat on a stump a while, then walked 25m east."

#### • PT #1: Draw a picture!

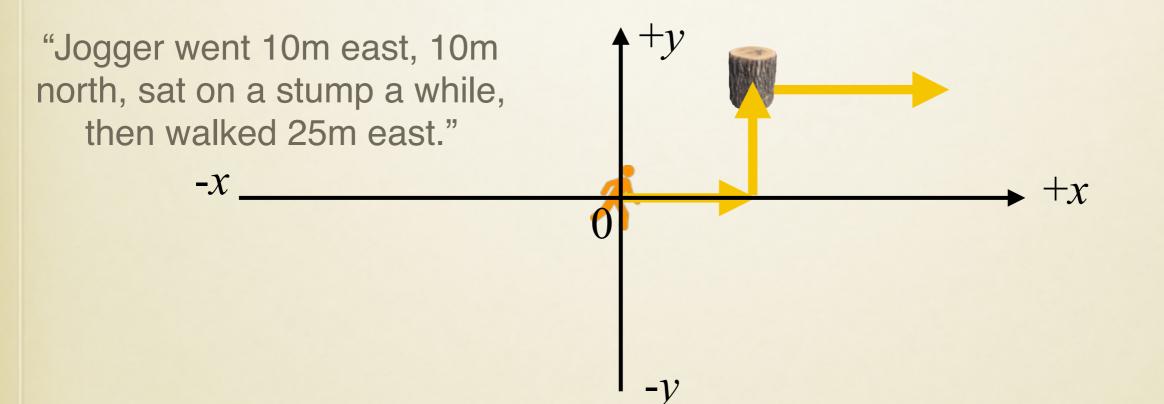
"Jogger went 10m east, 10m north, sat on a stump a while, then walked 25m east."



- PT #1: Draw a picture!
- PT #2: Use (and LABEL) a coordinate system.



- PT #1: Draw a picture!
- PT #2: Use (and LABEL) a coordinate system.



The direction of these arrows is important for setting up problems and **may** affect the sign of your variables and/or answers (will see example soon)

Ex: Car initially parked 3.0 m to right of house, drives around the block, ends up 5.0 m to left of house. Find the displacement of the car.

This one's easy, but let's practice pro tips!

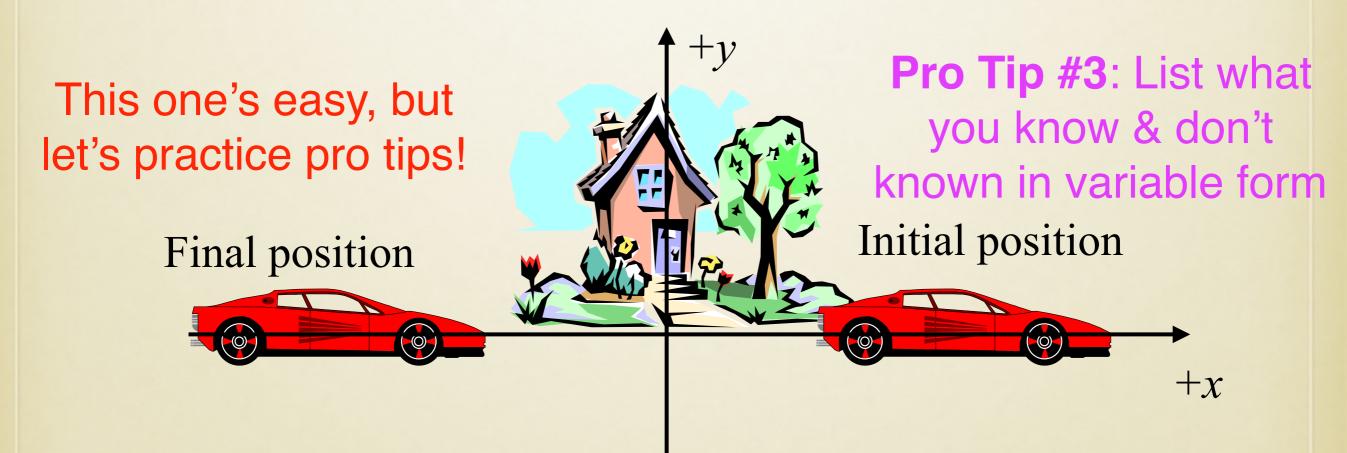
Ex: Car initially parked 3.0 m to right of house, drives around the block, ends up 5.0 m to left of house. Find the displacement of the car.

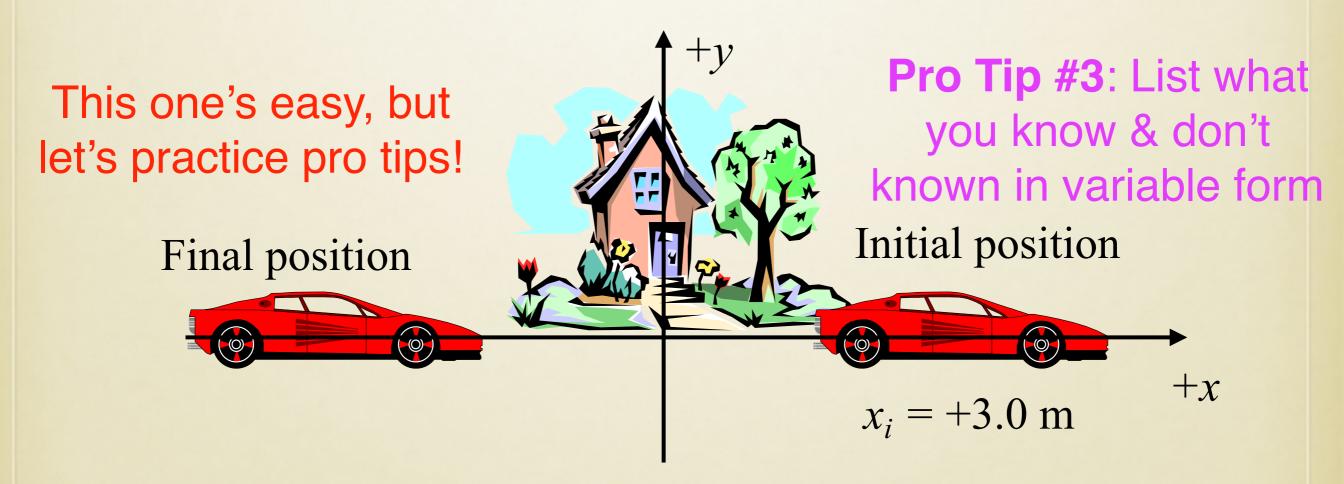
This one's easy, but let's practice pro tips!

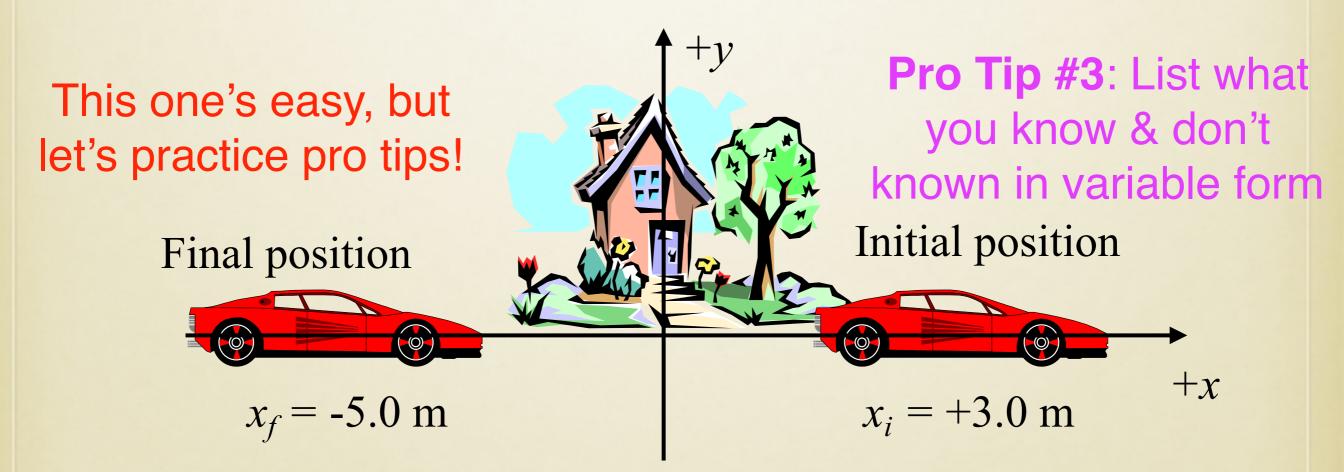
**Final** position



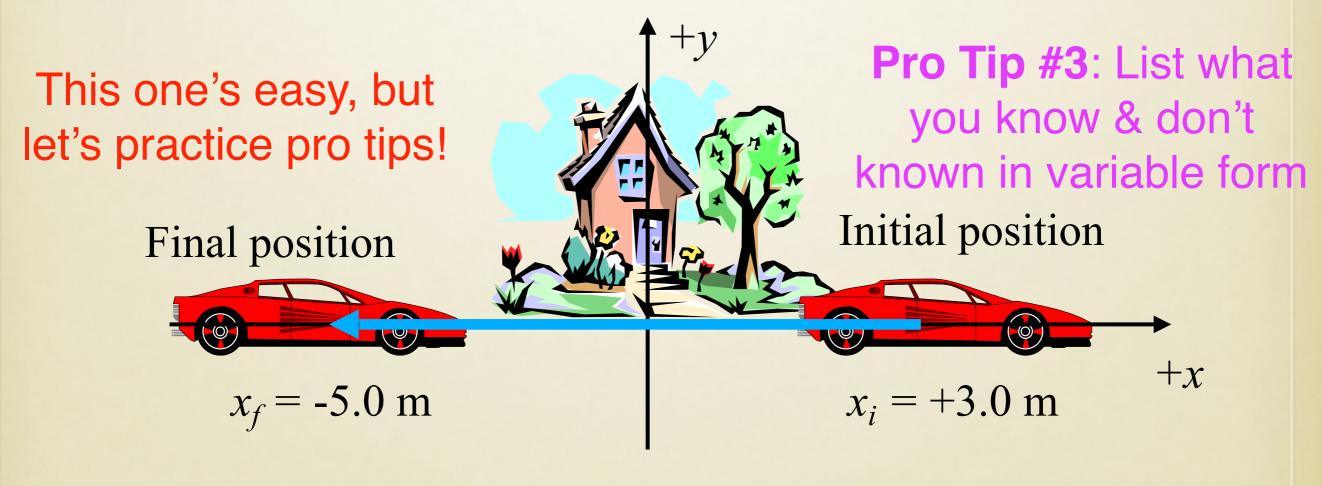






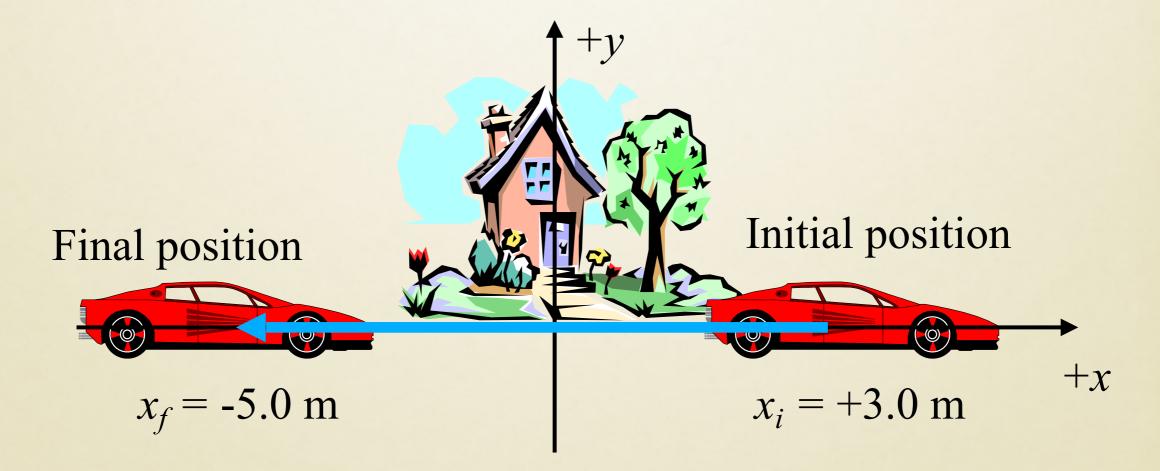


Ex: Car initially parked 3.0 m to right of house, drives around the block, ends up 5.0 m to left of house. Find the displacement of the car.

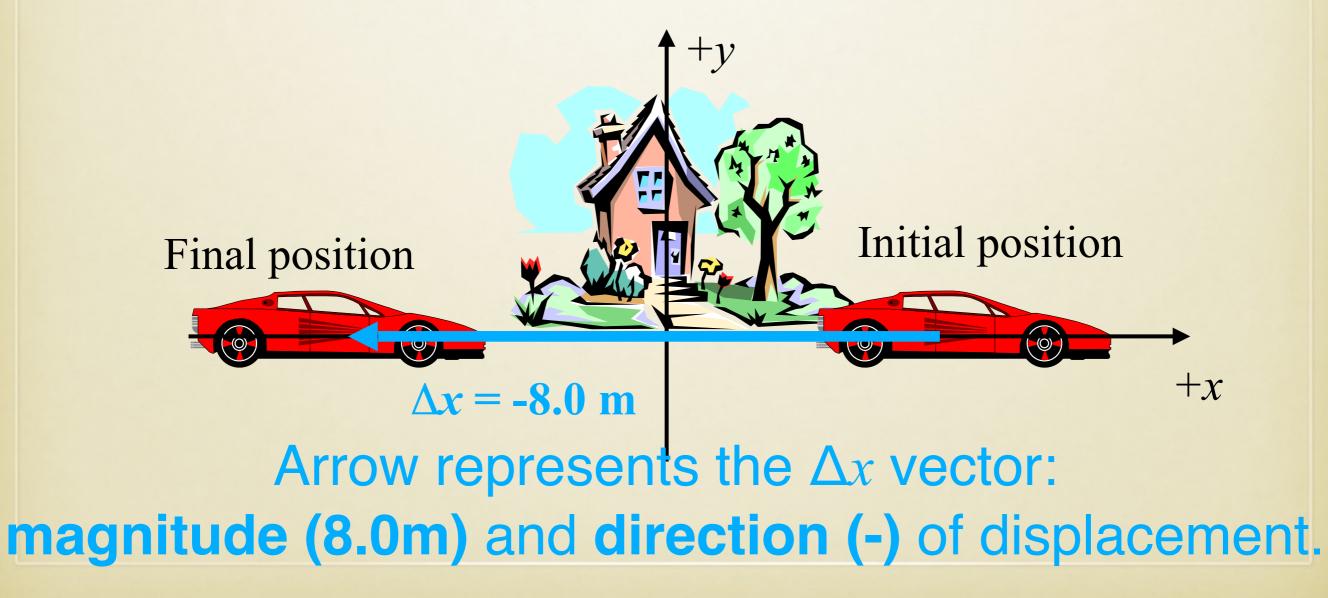


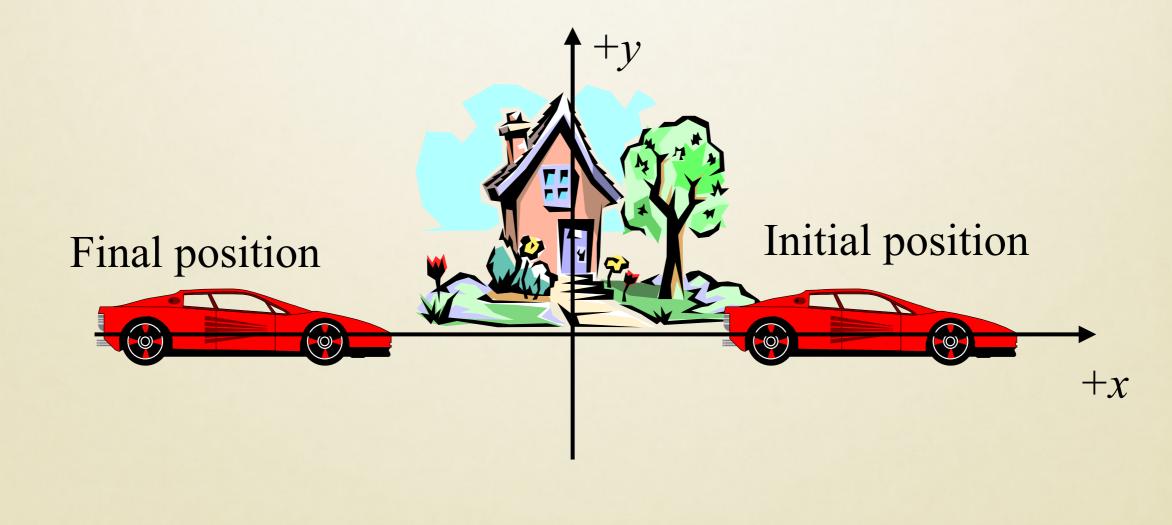
 $\Delta x = ?$ 

Ex: Car initially parked 3.0 m to right of house, drives around the block, ends up 5.0 m to left of house. Find the displacement of the car.

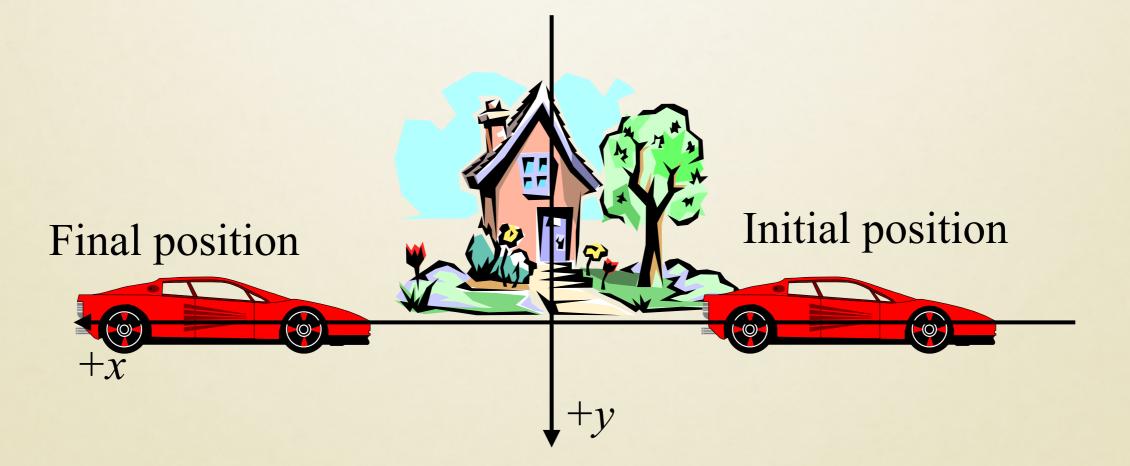


 $\Delta x = -5.0 \text{ m} - (+3.0 \text{ m}) = -8.0 \text{ m}$ 



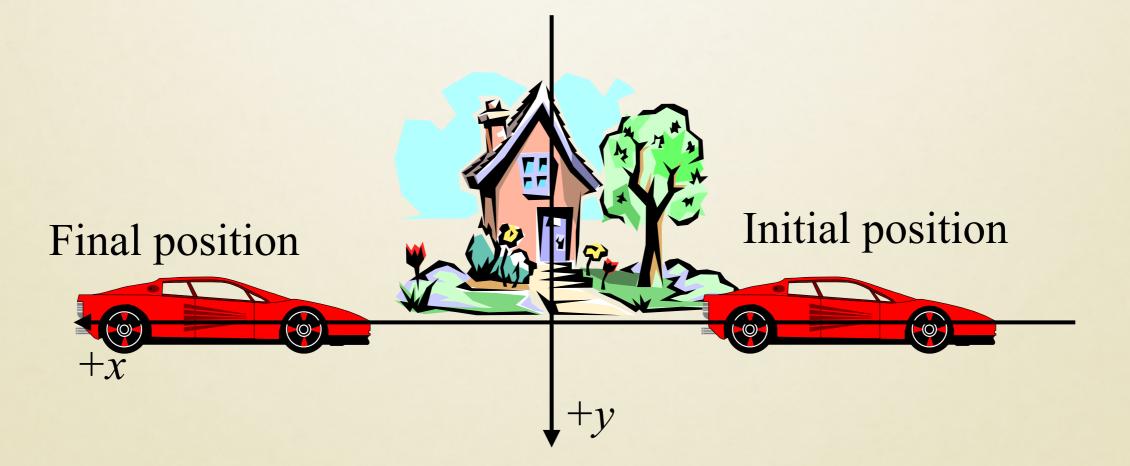


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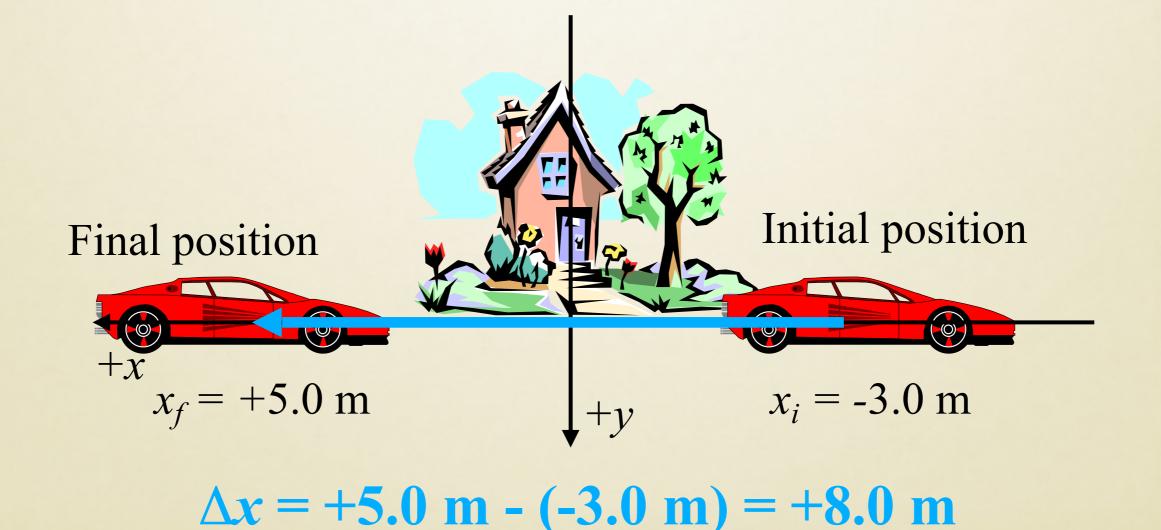


#### Write your knowns and unknowns!

Ex: Car initially parked 3.0 m to right of house, drives around the block, ends up 5.0 m to left of house. Find the displacement of the car.



#### Write your knowns and unknowns!



Many people struggle with signs! Ask yourself after defining each variable:

Is the sign consistent with what direction I've called positive?

Up and right are usually positive! (particularly in WebAssign unless explicitly stated in the problem)

Definition: velocity is displacement per unit time

$$\overline{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

SI units: m/s

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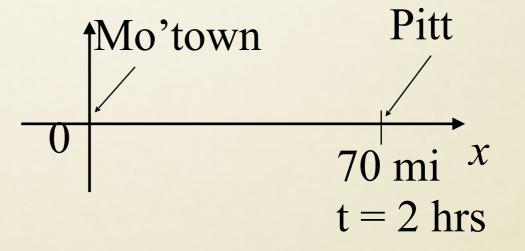
Ex: Go to Pittsburgh in 2 hrs, back in Morgantown 3 hrs after leaving

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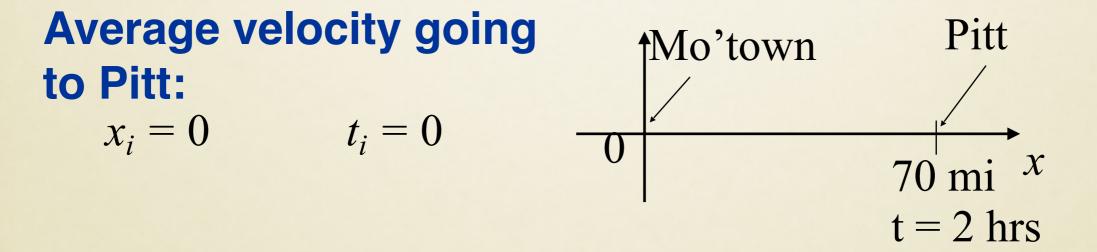
Average velocity going to Pitt:



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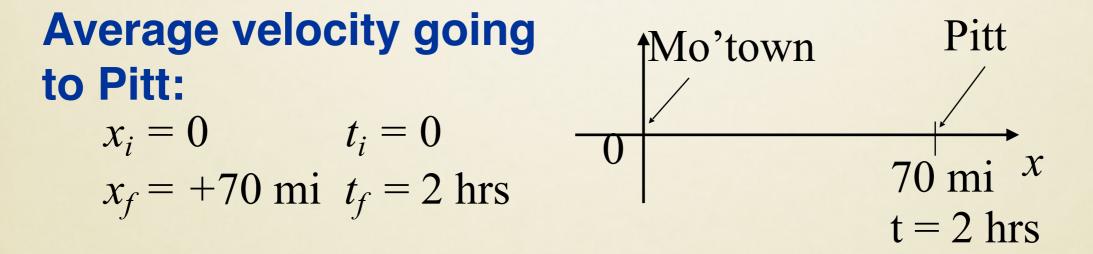
Ex: Go to Pittsburgh in 2 hrs, back in Morgantown 3 hrs after leaving



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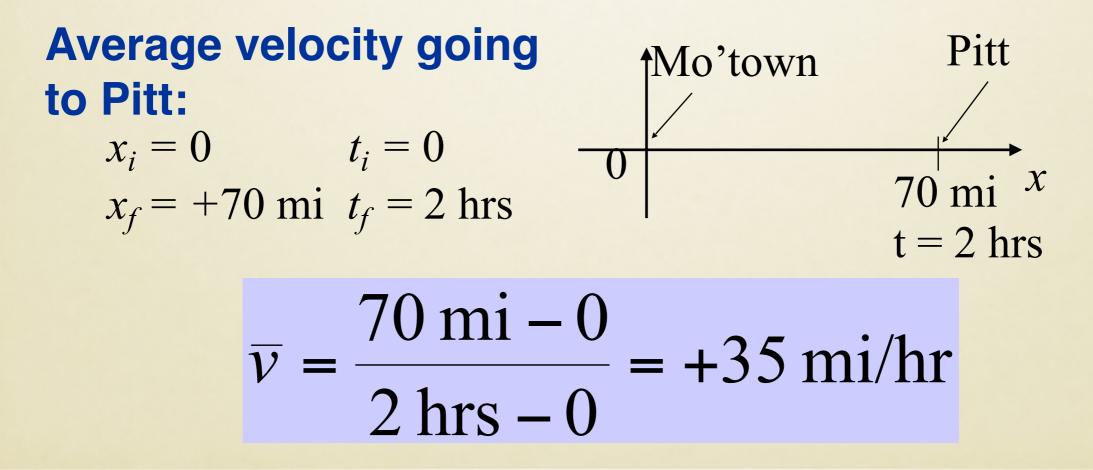
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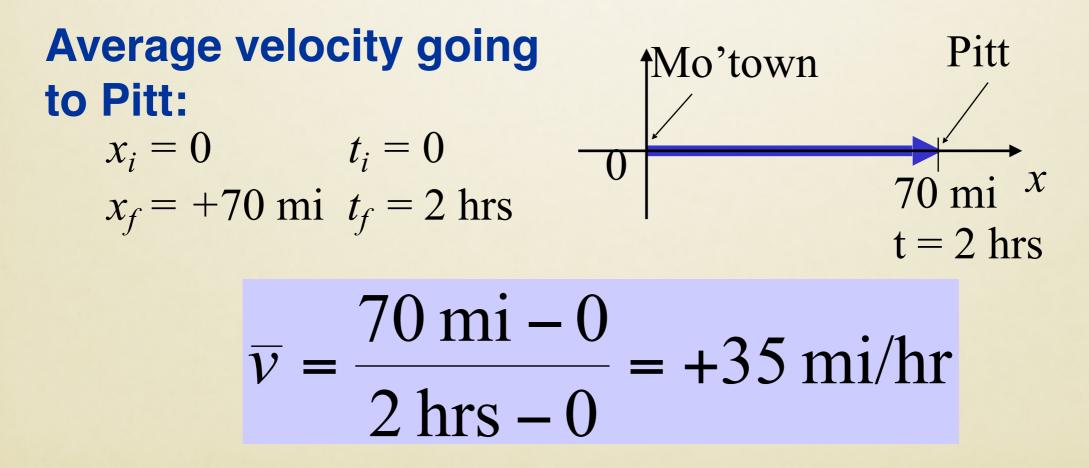
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 SI units: m/s

Ex: Go to Pittsburgh in 2 hrs, back in Morgantown 3 hrs after leaving

#### **Average velocity coming back from Pitt?**

Average velocity of round trip?

If you finish those: Average speed (scalar!) of round trip?

Definition: velocity is displacement per unit time

$$\overline{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

SI units: m/s

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Average velocity coming back from Pitt:

Definition: velocity is displacement per unit time

$$\overline{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$
 SI units: m/s

Ex: Go to Pittsburgh in 2 hrs, back in Morgantown 3 hrs after leaving

Average velocity coming back from Pitt:

$$\overline{v} = \frac{0 - 70 \text{ mi}}{3 \text{ hrs} - 2 \text{ hrs}} = -70 \text{ mi/hr}$$

Definition: velocity is displacement per unit time

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$
 SI units: m/s

Ex: Go to Pittsburgh in 2 hrs, back in Morgantown 3 hrs after leaving

Average velocity coming back from Pitt:

$$\overline{v} = \frac{0 - 70 \text{ mi}}{3 \text{ hrs} - 2 \text{ hrs}} = -70 \text{ mi/hr}$$

Average velocity of round trip:

Definition: velocity is displacement per unit time

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$
 SI units: m/s

Ex: Go to Pittsburgh in 2 hrs, back in Morgantown 3 hrs after leaving Mo'town Pitt

70 m

Average velocity coming back from Pitt:

$$\overline{v} = \frac{0 - 70 \text{ mi}}{3 \text{ hrs} - 2 \text{ hrs}} = -70 \text{ mi/hr}$$

Average velocity of round trip:

$$\overline{v} = \frac{0-0}{3\,\mathrm{hrs}-0} = 0$$

Definition: velocity is displacement per unit time

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$
 SI units: m/s  
Ex: Go to Pittsburgh in 2 hrs, back in Morgantown 3 hrs after leaving  
$$\frac{1}{0} \underbrace{\int_{0}^{\text{Mo'town}} \frac{1}{\sqrt{10} \text{ mi}} x}_{70 \text{ mi}}$$
  
Average velocity coming  
back from Pitt:  
$$\overline{v} = \frac{0 - 70 \text{ mi}}{3 \text{ hrs} - 2 \text{ hrs}} = -70 \text{ mi/hr}$$
  
Average velocity of round  
trip:  
$$\overline{v} = \frac{0 - 0}{3 \text{ hrs} - 0} = 0$$

## **Instantaneous** Velocity

- Instantaneous velocity is velocity at a particular instant.
- Only use the average velocity when asked for "average."

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Will discuss this difference more next lecture.

## Acceleration

Average acceleration = change in velocity/time

$$\overline{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

SI Units:  $m/s/s = m/s^2$ 

## Acceleration

Average acceleration = change in velocity/time

$$\frac{-}{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

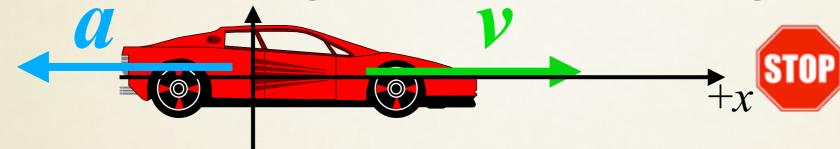
Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$
 SI Units:  
m/s/s = m/s<sup>2</sup>

The sign of acceleration indicates which direction its velocity changes. Positive acceleration means speeding up when moving in the positive x direction OR slowing down when moving in the negative x direction.

# Signs of acceleration

A car slowing down at a stop sign



A bullet hitting a wall

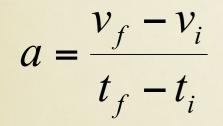
# +x

#### Sprinter out of the blocks



$$t_f = t$$
  $t_i = 0$  "t at time zero"  
 $x_f = x$   $x_i = x_o$  "location at time zero"  
 $v_f = v$   $v_i = v_o$  "velocity at time zero"

$$t_f = t$$
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 $x_f = x$   $x_i = x_o$  "location at time zero"  
 $v_f = v$   $v_i = v_o$  "velocity at time zero"

$$a = \frac{v_f - v_i}{t_f - t_i} \implies a = \frac{v - v_o}{t}$$

Notation:

$$= t$$
 $t_i = 0$ "t at time zero" $= x$  $x_i = x_o$ "location at time zero" $= v$  $v_i = v_o$ "velocity at time zero"

$$a = \frac{v_f - v_i}{t_f - t_i} \implies a = \frac{v - v_o}{t} \implies v = v_o + at$$

 $t_f$ 

 $X_f$ 

 $\mathcal{V}_f$ 

Notation:

$$a = \frac{v_f - v_i}{t_f - t_i} \implies a = \frac{v - v_o}{t} \implies v = v_o + at$$

 $t_f$ 

 $X_f$ 

 $\mathcal{V}_f$ 

 $v_{avg} = \frac{x_f - x_i}{t_f - t_i}$ 

Notation:

$$t_f = t$$
  $t_i = 0$  "t at time zero"  
 $x_f = x$   $x_i = x_o$  "location at time zero"  
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 $X_f$ 

 $\mathcal{V}_{f}$ 

$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} \longrightarrow v_{avg} = \frac{x - x_o}{t}$$

$$t_f = t$$
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$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} \longrightarrow v_{avg} = \frac{x - x_o}{t} \longrightarrow x = x_o + v_{avg} t$$

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$$v_{avg} = \frac{v + v_o}{2}$$

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$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} \qquad v_{avg} = \frac{x - x_o}{t} \qquad x = x_o + v_{avg}t$$

Similar derivations lead to more equations:

$$v_{avg} = \frac{v + v_o}{2}$$

Notation:

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Similar derivations lead to more equations:

$$v_{avg} = \frac{v + v_o}{2}$$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

Notation:

$$t_f = t$$
  $t_i = 0$  "t at time zero"  
 $x_f = x$   $x_i = x_o$  "location at time zero"  
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Similar derivations lead to more equations:

$$v_{avg} = \frac{v + v_o}{2}$$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

$$v_{avg} = \frac{v - v_o}{2}$$

$$v = v_o + at$$

$$v^2 = v_o^2 + 2a\Delta x$$

$$v_{avg} = \frac{v - v_o}{2}$$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

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Pro Tip #3: List what you know and need to know in variable form

$$v = v_o + at \qquad v^2 = v_o^2 + 2a\Delta x$$

$$v_{avg} = \frac{v - v_o}{2} \qquad \qquad \Delta x = v_o t + \frac{1}{2} a t^2$$

Pro Tip #3: List what you know and need to know in variable form

1 equation with one unknown is solvable.

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$$v_{avg} = \frac{v - v_o}{2} \qquad \qquad \Delta x = v_o t + \frac{1}{2} a t^2$$

Pro Tip #3: List what you know and need to know in variable form

- 1 equation with one unknown is solvable.
- 2 equations with two unknowns is solvable.

$$v = v_o + at \qquad v^2 = v_o^2 + 2a\Delta x$$

$$v_{avg} = \frac{v - v_o}{2} \qquad \qquad \Delta x = v_o t + \frac{1}{2} a t^2$$

Pro Tip #3: List what you know and need to know in variable form

- 1 equation with one unknown is solvable.
- 2 equations with two unknowns is solvable.

Pro Tip # 4: Practice helps you pick best formulas!

The speed of a nerve impulse in the human body is about 100 m/s. If you accidentally stub your toe in the dark, **estimate** the time it takes the nerve impulse to travel to your brain.



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Draw a picture and list knowns and unknowns

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Average velocity = 100 m/s = displacement / time

The speed of a nerve impulse in the human body is about 100 m/s. If you accidentally stub your toe in the dark, **estimate** the time it takes the nerve impulse to travel to your brain.

Draw a picture and list knowns and unknowns Average velocity = 100 m/s = displacement / time

Change in time =  $\Delta t = \Delta x/v = \sim 2 \text{ m} / 100 \text{ m/s}$ 

The speed of a nerve impulse in the human body is about 100 m/s. If you accidentally stub your toe in the dark, **estimate** the time it takes the nerve impulse to travel to your brain.

Draw a picture and list knowns and unknowns Average velocity = 100 m/s = displacement / time

Change in time =  $\Delta t = \Delta x/v = ~2 \text{ m} / 100 \text{ m/s}$ = 0.02 s or 20 milliseconds

## Problems inside problems

Might need to break down problem into smaller pieces! Solve in sequence.

A rocket ship is capable of accelerating at a rate of 0.60 m/s<sup>2</sup>. How long does it take for it to get from going 55 mi/h to going 60 mi/h?

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Draw a picture and list knowns and unknowns Want:  $\Delta t$  Know:  $v_o$ ,  $v_f$ , a

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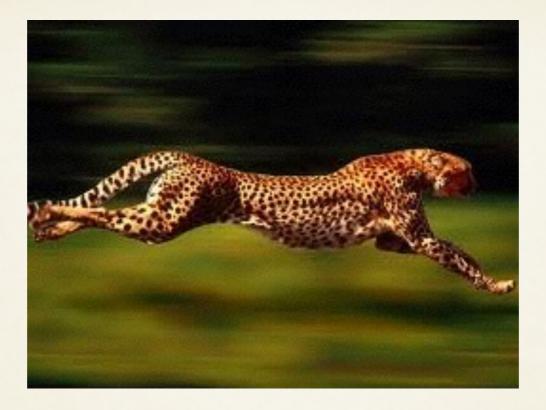
 $v = v_o + a \Delta t$  rearrange:  $\Delta t = (v - v_o)/a$ 

A rocket ship is capable of accelerating at a rate of 0.60 m/s<sup>2</sup>. How long does it take for it to get from going 55 mi/h to going 60 mi/h?

Draw a picture and list knowns and unknowns Want:  $\Delta t$  Know:  $v_o$ ,  $v_f$ , a

 $v = v_o + a \Delta t$  rearrange:  $\Delta t = (v - v_o)/a$ 

Will need to convert mi/h to what? 1 mile = 1609 m



While chasing its prey in a short sprint, a cheetah starts from rest and runs 45 m in a straight line, reaching a final speed of 72 km/h. (a) Determine the cheetah's average acceleration during the short sprint, and (b) find its displacement at t = 3.5s.

# **Problem Solving Pro-tips**

- 1. Draw a picture!
- 2. Use and label your reference frame.
- 3. List what you KNOW and DON'T KNOW in variable form.
- 4. Practice helps you pick best formulas!