Gravitational Waves

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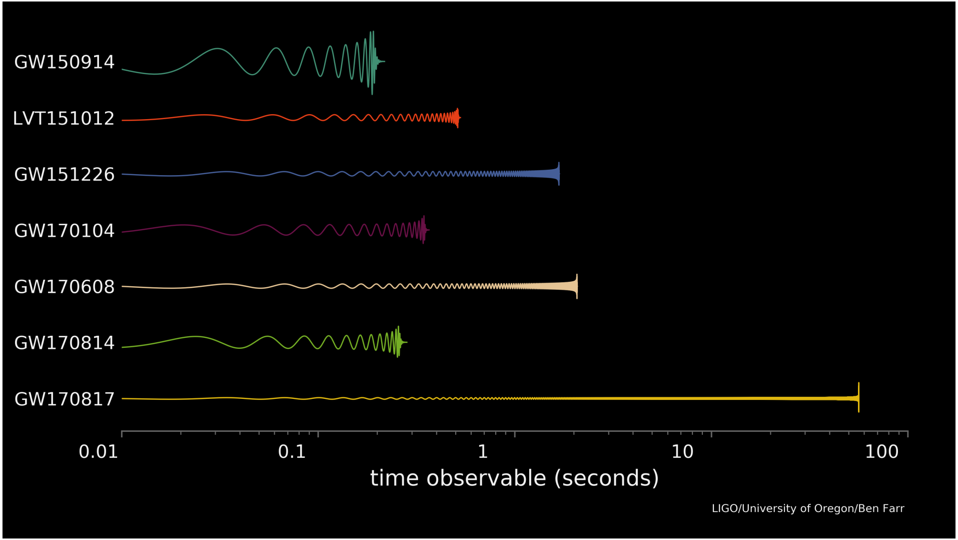
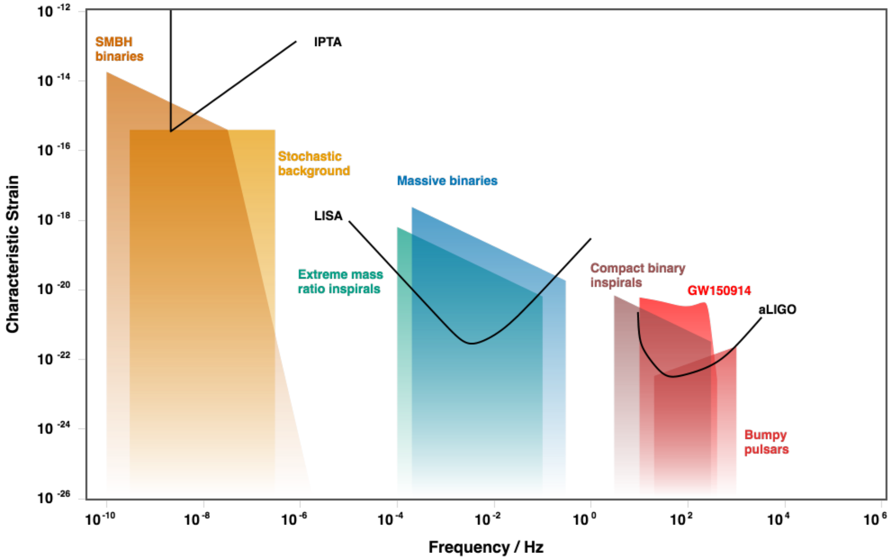
CENTER FOR GRAVITATIONAL WAVES AND COSMOLOGY

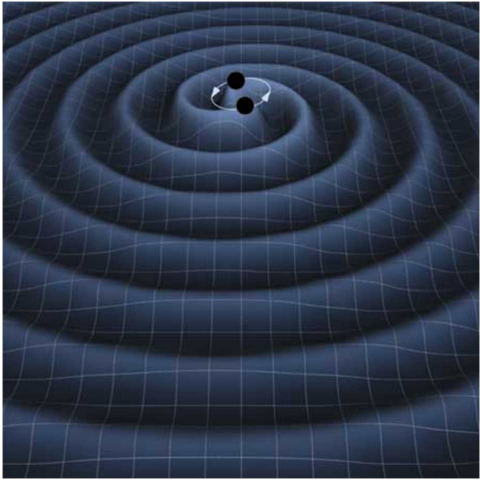
LIGO detected GWs from **stellar mass** compact binaries

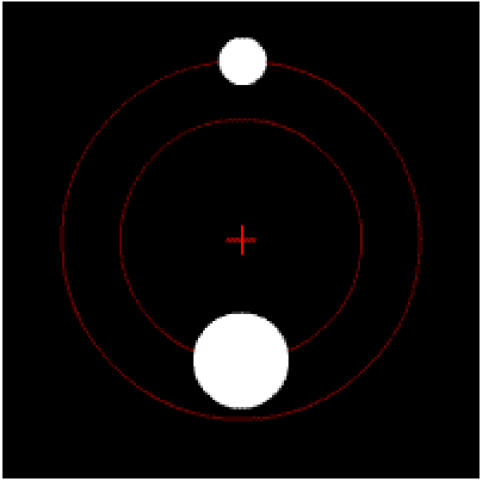


There's a whole spectrum of GWs



GWs are generated by accelerating mass

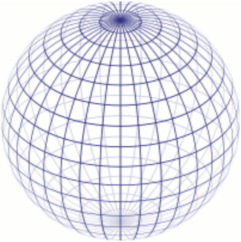


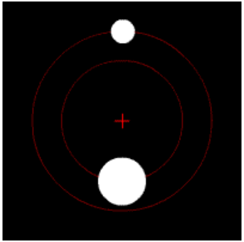


well... GWs are generated by changing mass quadrupole moments

mass quadrupole moment is a 3x3 matrix:

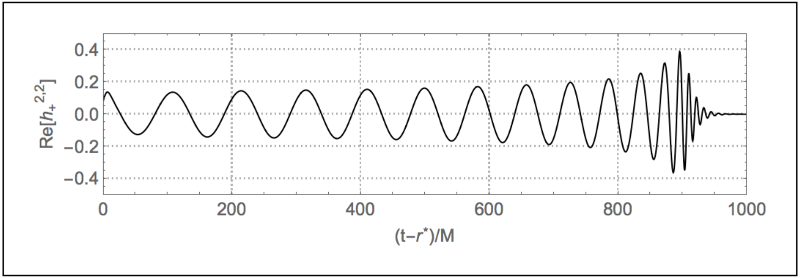
$$I_{ij}(t) = \int \rho(t,x)\, x^i x^j \mathrm{d}^3 x$$





$$\boldsymbol{h} \sim \ddot{\boldsymbol{I}}$$

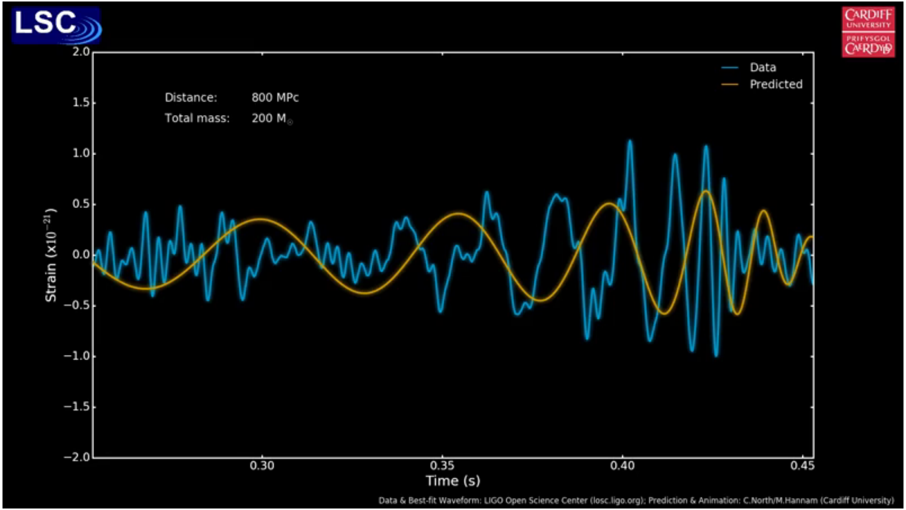
We can learn about the source system by measuring the GWs



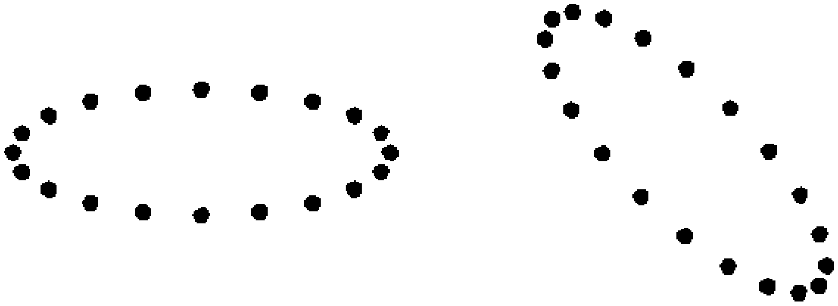
for a binary system, GW frequency is proportional to orbital frequency

$$\omega_{\rm gw} = 2\omega_{\rm orbit}$$

Fit a predicted waveform to data to measure

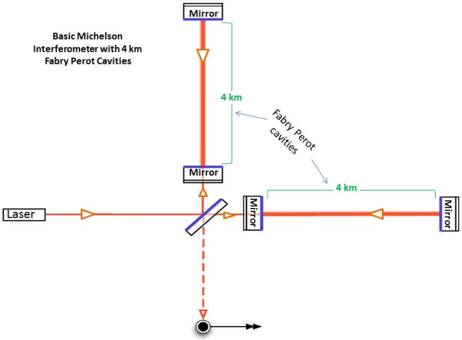


Gravitational waves cause the distance between free floating particles to change

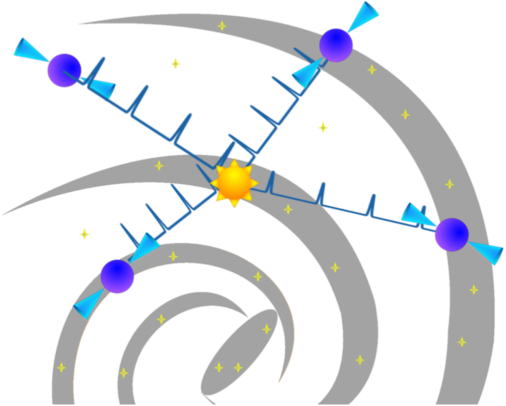


We measure GWs by monitoring the distance between objects

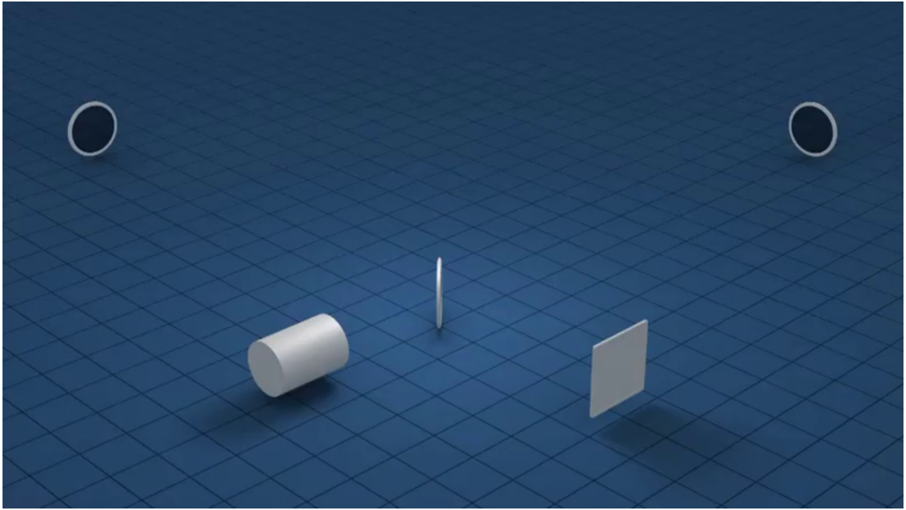
LIGO:



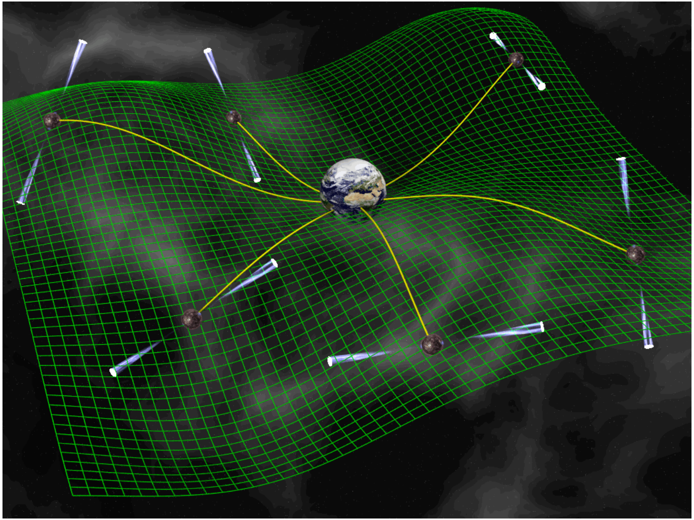
Pulsar Timing Arrays:



LIGO measures tiny changes in the distance between mirrors



PTAs monitor the change in distance between the Earth and many millisecond pulsars



lets do some math!

to do GW calculations, remember Special Relativity

- set speed of light $c = 1$. Measure time in distance units: $t = ct$
- 4-vectors: $\vec{s} \rightarrow (t, x, y, z)$
- new 4D dot product (now with minus sign!)

  e.g. invariant interval:

$$\Delta s^2 = \Delta\vec{s} \cdot \Delta\vec{s} = -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

- Minkowski metric:

$$\boldsymbol{\eta} \rightarrow \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

gravitational waves are perturbations of spacetime

$$\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}$$

- spacetime metric
- Minkowski metric (flatspace)
- small perturbation ($|h| \ll 1$)

all are 4x4 matrices (t, x, y, z)

$$\boldsymbol{\eta} \rightarrow \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In curved space we use $\boldsymbol{g}$ for the dot product!

gravitational waves satisfy the wave equation

$$\Box \boldsymbol{h} = 0$$

solutions:

$$\boldsymbol{h} = \boldsymbol{A} \exp(\mathrm{i}\, \vec{k} \cdot \vec{s})$$

- wave operator, a.k.a. d'Alembertian:

$$\Box = \vec{\partial} \cdot \vec{\partial} = -\partial_t{}^2 + \partial_x{}^2 + \partial_y{}^2 + \partial_z{}^2$$

- wave vector defines frequency and propagation direction:

$$\vec{k} \to (\omega, k_x, k_y, k_z)$$

- tensor amplitude: $\boldsymbol{A}$

gravitational waves travel at the speed of light

$$
\begin{aligned}
\Box \boldsymbol{h} &= 0 \\
\Box \left[\boldsymbol{A} \exp(\mathrm{i}\, \vec{k} \cdot \vec{s}) \right] &= \\
\boldsymbol{A}\, \Box \left[\exp(\mathrm{i}\, \vec{k} \cdot \vec{s}) \right] &= \\
-(\vec{k} \cdot \vec{k})\, \boldsymbol{A}\, \exp(\mathrm{i}\, \vec{k} \cdot \vec{s}) &= 0
\end{aligned}
$$

$$\vec{k} \cdot \vec{k} = 0$$

GWs propagate in the **light-like** direction (along **null geodesics**)

assume wave propagates in $\hat{z}$ direction

$$\vec{k} \to (\omega, 0, 0, k) = (\omega, 0, 0, \omega)$$

$$\boldsymbol{A} \to \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{h}(\vec{s}) = \boldsymbol{h}(t, z) = \boldsymbol{A} \exp\left[\mathrm{i}\,\omega(z - t)\right]$$

For calculations in the spacetime with GW we us $\boldsymbol{g}$

$$\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}$$

$$\boldsymbol{g} \to \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 + h_+ \exp\left[\mathrm{i}\,\omega(z-t)\right] & h_\times \exp\left[\mathrm{i}\,\omega(z-t)\right] & 0 \\ 0 & h_\times \exp\left[\mathrm{i}\,\omega(z-t)\right] & 1 - h_+ \exp\left[\mathrm{i}\,\omega(z-t)\right] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{a} \cdot \vec{b} = \sum_{\mu,\nu} a^\mu g_{\mu\nu} b^\nu$$

What is the proper separation ($|\Delta s|$) between two points in the presence of GWs?

Choose one point to be the origin $\vec{s}_0 \rightarrow (0,0,0,0)$. Do the calculation 5 times, using each of the following second points:

$$\vec{s}_t \rightarrow (\tau,0,0,0) \qquad \vec{s}_x \rightarrow (0,\ell,0,0)$$

$$\vec{s}_y \rightarrow (0,0,\ell,0)$$

$$\vec{s}_\star \rightarrow \frac{1}{\sqrt{2}}(0,\ell,\ell,0) \qquad \vec{s}_z \rightarrow (0,0,0,\ell)$$