Projectile Motion
Problem-Solving

**Last year’s exam equation sheet**

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**POTENTIALLY USEFUL INFORMATION:**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m = 3.281 ft</td>
<td>1 mile = 1609 m</td>
</tr>
<tr>
<td>( v_{xy} = (v_x - v_t)(t - t_0) )</td>
<td>( a_{xy} = (v_y - v_t)(t - t_0) )</td>
</tr>
<tr>
<td>( v = v_0 + \Delta t )</td>
<td>( \Delta x = v_y \cdot \Delta t + \frac{1}{2} \Delta a )</td>
</tr>
<tr>
<td>( v_x = v_x \cdot \cos\theta )</td>
<td>( v_y = v_y \cdot \sin\theta )</td>
</tr>
<tr>
<td>1 kg = 2.2 pounds</td>
<td>quadratic: ( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</td>
</tr>
</tbody>
</table>

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What are you getting stuck on in problem-solving?

**PRACTICE MORE!**
And come talk to me or the TAs.

You won’t do well if you wait then cram.

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**Practice Exams**

- This weekend, take this like a real test.
  Did you pass? Where did you get stuck?

- Posting answer keys next Wednesday.

- Your exam:
  - 20 questions
  - 3 hours
  - ~25-50% conceptual, ~50-75% calculations

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**Exam logistics**

- **INFORM ME OF CONFLICTS BY 12 SEPT.**
  otherwise you might not take exam

- September 19, 7-10 pm

- Bring:
  - Pencil, eraser, non-graphing calculator

- Location: White Hall
  - **Last name A-L: G09 (main building entrance)**
  - **Last name M-Z: B51 (regular classroom)**

- If your phone is seen, you will be asked to leave.
Note: WebAssign Things

• “Your answer is within 10%”
  ➔ Round off at more digits in calculation (at least 3-4).
  ➔ Otherwise you did the calculation wrong.
• Answer keys can be accessed after due date.
• If you’ve missed an assignment:
  • Don’t access answer key.
  • Request automatic extension.

Questions?

Warm-up: Graphing

What’s the sign of the displacement between 30 and 45 seconds?

A. negative
B. zero
C. positive
D. not enough info to determine

Answer: A

Last time you guys bombed on graphing. Let’s practice.

Warm-up: Graphing

What’s the sign of the displacement between 30 and 45 seconds?

A. negative
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C. positive
D. not enough info to determine
Warm-up: Graphing

What's the average velocity between 30 and 45 seconds?

A. 1.3 m/s  
B. 13 m/s  
C. 15 m/s  
D. 20 m/s  
E. None of the above

Graphing again!

What's the average velocity between 30 and 45 seconds?

A. 1.3 m/s  
B. 13 m/s  
C. 15 m/s  
D. 20 m/s  
E. None of the above

Symbolic Reasoning

If you toss a ball upward with a certain initial speed, it falls freely and reaches a maximum height $h$. By what factor must you increase the initial speed of the ball for it to reach a maximum height $4h$?

$$v = v_0 + at$$

$$\Delta x = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Note: it's a symbolic/conceptual problem, but you can still use problem solving tips (and if you want, fake numbers!)

Symbolic Reasoning

If you toss a ball upward with a certain initial speed, it falls freely and reaches a maximum height $h$. By what factor must you increase the initial speed of the ball for it to reach a maximum height $4h$?

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A. 2  
B. 3  
C. 4  
D. 8  
E. 16

Note: it's a symbolic/conceptual problem, but you can still use problem solving tips (and if you want, fake numbers!)
Tossing something from a roof
(5 examples with increasing difficulty)

A ball is launched from the edge of a 15.0m tall building at 16 m/s at an angle of 60 degrees from the horizontal.

1. **How much time** does it take to fall?
2. **How far from the base of the cliff** does it hit the ground? (Need the time first)
3. **How fast** it is moving vertically when it hits the ground? (y component of final velocity)
4. What is the magnitude of its velocity when it hits the ground?
5. What is the angle that it hits the ground from the horizontal?

Tossing something from a roof
(5 examples with increasing difficulty)

A ball is launched from the edge of a 15.0m tall building at 16 m/s at an angle of 60 degrees from the horizontal.

What is the magnitude of its velocity right before it hits the ground?

What is the angle it hits the ground in reference to horizontal?

\[
\begin{align*}
\Delta x &= v_0 t + \frac{1}{2} at^2 \\
v &= v_0 + at \\
v^2 &= v_0^2 + 2a\Delta x
\end{align*}
\]

Vector math!

Projectile motion or that trig mountain problem?

[note: this clicker not graded]

A. More projectile motion practice

B. Trig mountain

Projectile Motion

A penguin runs horizontally off the top of an iceberg at 3 m/s and hits the water at a distance of 10m. How tall is the iceberg?
Projectile Motion

A penguin runs horizontally off the top of an iceberg at 3 m/s and hits the water at a distance of 10 m. How tall is the iceberg?

Does this problem require analysis of horizontal or vertical movement?
A. Vertical  
B. Horizontal  
C. Both

Remember!

Treat X and Y movements separately until asked for actual speed/velocity!
(or total velocity, net velocity, magnitude of velocity)

Remember!

The time will be the same for x and y parts.
If you don’t have enough information for x or y components, solve for time and reassess what you can determine.

Remember!

If you get confused about variables, sometimes it really helps to rewrite your motion equations in terms of x and y components.

Try writing them out before projectile motion problems…

\[ v = v_0 + at \]
\[ v_x = v_{0x} + a_xt \]
\[ v_y = v_{0y} + a_yt \]
\[ \Delta x = v_{0x}t + \frac{1}{2} a_xt^2 \]
\[ \Delta x = v_{0x}t + \frac{1}{2} a_xt^2 \]
\[ \Delta y = v_{0y}t + \frac{1}{2} a_yt^2 \]
\[ v_x^2 = v_{0x}^2 + 2a_x\Delta x \]
\[ v_y^2 = v_{0y}^2 + 2a_y\Delta y \]
**Remember!**

If you get confused about variables, sometimes it really helps to rewrite your motion equations in terms of x and y components.

Try writing them out before projectile motion problems...

\[
\begin{align*}
\mathbf{a}_x &= 0 \text{ m/s}^2 \\
\mathbf{a}_y &= -g = -9.8 \text{ m/s}^2
\end{align*}
\]

\[
\begin{align*}
v &= v_0 + at \\
v_x &= v_{x0} \\
v_y &= v_{y0} - gt \\
\Delta x &= v_{x0}t + \frac{1}{2}at^2 \\
\Delta y &= v_{y0}t - \frac{1}{2}gt^2 \\
v_x^2 &= v_{x0}^2 \\
v_y^2 &= v_{y0}^2 - 2g\Delta y
\end{align*}
\]

* ONLY IF you define +y as up (like we usually do!)

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A penguin runs horizontally off the top of an iceberg at 3 m/s and hits the water at a distance of 10m. How tall is the iceberg?

We don't have enough info to solve for \(\Delta y\)!

So solve for time in the x-dimension.

Now you can solve for \(\Delta y\)!

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That trig problem...

A woman measures the angle of elevation of a mountaintop. Suppose the mountain height is \(y\), the woman's original distance from the mountain is \(x\), and the angle of elevation she measures from the horizontal to the top of the mountain is \(\theta\). If she moves a distance \(d\) closer to the mountain and measures an angle of elevation \(\phi\), find a general equation for the height of the mountain \(y\) in terms of \(d\), \(\phi\), and \(\theta\), neglecting the height of her eyes above the ground.