Centripetal Acceleration and Newtonian Gravitation

Today we’ll be doing one more new concept before the test on Wednesday.

Again, the test is this Wednesday and your movie assignment is due NEXT WEEK, THE 20TH!
What path do the longest sparks take after they leave the wand?

Centripetal Acceleration and Newtonian Gravitation
Reminders: 15 March 7-10pm, Exam 2! Eiesland G24.
Movie review project draft 1 due in 1 week (20 Mar)!

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Again, the test is this Wednesday and your movie assignment is due NEXT WEEK, THE 20TH!
We talked before break about how we can define rotational motion through angular displacement, velocity, and acceleration much like we can the linear quantities - but now we’re talking about spin rates. Anything moving in a curved path is undergoing angular motion.
Today, we’re going to start applying these to things moving around a curve and talk about the forces that govern that motion. We'll also apply this to gravity. My field of study is gravity so I’m going to be real here and say that Newtonian Gravity that we’ll discuss today is an ok approximation for how gravity works (there are a few fully descriptive theories of gravity out there, by the way, but the most broadly used is general relativity, which was developed by Einstein about 100 years ago).
A rap about your physics misconceptions.

Do you remember the conceptual pre-test? This lecture covers the stuff where you guys were the wrong-est.

For the conceptual pre-test, centripetal motion is where you guys had the worst misconceptions.
You’re looking down at a frictionless channel anchored to a frictionless horizontal table top. Ignore air resistance. A ball is shot at high speed into the channel at $P$ and exits at $R$.

Consider the following forces:
1. downward force of gravity.
2. a force exerted by the channel, in the direction from $Q$ to $O$.
3. a force in the direction of motion.
4. a force pointing from $O$ to $Q$.

Which of the above forces is (are) acting on the ball when it is at point $Q$?

- A. 1 only.
- B. 1 and 2.
- C. 1 and 3.
- D. 1, 2, and 3.
- E. 1, 3, and 4.

Answer: B.

For real: on the FCI only 7/170 of you got this question right! You would have done better for it if you guessed randomly! That means almost all of you had to have thought about the question and chosen the wrong answer: thus have serious misconceptions about circular motion. Today we will fix that!
Uniform Circular Motion

Remember Newton’s first law of motion?

Things will travel in **same direction** at **same speed** unless an external force is applied to the object.
Uniform Circular Motion

Remember Newton’s first law of motion?

Spinning a ball on a string at constant angular velocity \( \omega \) (\( \alpha = 0 \)).

*Instantaneous velocity vector for this motion shown in green.*

Central to our understanding of centripetal motion is understanding the concepts and details of uniform circular motion. Look at what's happening here. Here's a ball on a string swinging around and around. This thing is going at constant angular velocity omega, and this ball is moving in a circular path. [For instance, swinging keys around a keychain] It does this because I'm constantly pulling on the string. BUT, things want to go in a straight line. On the slide, I'm showing the velocity vector of this ball. It's always **tangential** to the circle! This is because it is always wanting to continue in the direction it's going. [If I let go of my keys, they will fly off at a tangent to where they were spinning].

Whenever you see something moving ALONG AN ARC, I want you to picture the velocity vector like this. **TANGENTIAL** to that circular motion.
Uniform Circular Motion

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same direction at same speed

unless an external force is applied to the object.

Spinning a ball on a string

at constant angular

velocity $\omega$ ($a=0$).

*Instantaneous* velocity

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**Uniform Circular Motion**

Remember Newton’s first law of motion?

![Diagram of uniform circular motion]

Velocity **direction changes**, but **speed** (magnitude of velocity) **does not change**!

Spinning a ball on a string at constant angular velocity $\omega$ ($\alpha=0$).

OK, so we know that in circular motion, we know that the velocity vector changes as something travels in a circle. [See light board notes here]. At different points along this circle, the VELOCITY changes, but the speed does NOT CHANGE. How do you get a change in velocity but not a change in speed? Think about keys on string, or ball on a string, being swung around in a circle. I feel a tension in this string that’s maintained through the full loop. I know that I have to apply an inward force to keep the keys spinning.
Uniform Circular Motion

Remember Newton’s first law of motion?

Velocity direction changes, but speed (magnitude of velocity) does not change!

What forces are acting on the ball?

Spinning a ball on a string at constant angular velocity \( \omega \) \((\alpha=0)\).

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Uniform Circular Motion

Remember Newton’s first law of motion?

String maintains tension pulling ball inward: This causes centripetal acceleration!

Velocity direction changes, but speed (magnitude of velocity) does not change!

WHAT FORCES ARE ACTING ON THE BALL?

This inward pull, that tension, is what we call a centripetal force, which maintains a centripetal acceleration happens. Centripetal means inward-seeking… it’s what maintains circular motion. This force is applying an inward acceleration that changes the velocity vector’s direction. We can show this… [see light board notes here]
There is a centripetal force/acceleration on anything moving in a circular path!

Centripetal acceleration
\[ a_c = \frac{v^2}{r} = r\omega^2 \]

Centripetal force
\[ F_c = ma_c = \frac{mv^2}{r} = mr\omega^2 \]

In the hoop example, the hoop has a force on the ball. For the car, static friction and a normal force for a bank in the track apply inward force on the car.

Cool demos:
https://www.youtube.com/watch?v=ID0R43My4Co
Zelda spinning keys:
https://www.youtube.com/watch?v=bpFK2VCRHUs
More good explanations:
https://www.youtube.com/watch?v=Usg_IPT5gJQ
Tangential and Total acceleration

Previous slides were all about constant speed. What if we now add some acceleration in spin rate?

The girl here is speeding up the spin rate of the whirligig, changing its angular velocity. Thus she is giving it angular acceleration, which changes not only her angular velocity but also tangential velocity.
Tangential and Total acceleration

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[see light board notes on how to get total acceleration]

Just to remind you: applying a constant acceleration tangential to a circle increases the spin rate, hence we can relate this also to the angular acceleration—how rapidly the spin rate of the object is changing.
Tangential and Total acceleration

Previous slides were all about constant speed. What if we now add some acceleration in spin rate?

Recall from last lecture the relation:

\[ a_t = r \alpha \]

[see light board notes on how to get total acceleration]

Just to remind you: applying a constant acceleration tangential to a circle increases the spin rate, hence we can relate this also to the angular acceleration—how rapidly the spin rate of the object is changing.
Let’s try it.

A race car accelerates uniformly from a speed of 40 m/s to a speed of 60 m/s in 5s while travelling counterclockwise around a circular track of radius 400m. When the car reaches a speed of 50 m/s, find

a) the magnitude of the car’s centripetal acceleration.
b) the angular speed.
c) the magnitude of the tangential acceleration.
d) the magnitude of the total acceleration.

\[ v = v_0 + at \]
\[ \Delta x = v_0 t + \frac{1}{2} at^2 \]
\[ v^2 = v_0^2 + 2a\Delta x \]
\[ \omega = \omega_0 + \alpha t \]
\[ \Delta \theta = \omega t + \frac{1}{2} \alpha t^2 \]
\[ \omega^2 = \omega_0^2 + 2\alpha \Delta \theta \]

\[ v = r \omega \]
\[ a_t = r \alpha \]
\[ a_c = \frac{v^2}{r} = r\omega^2 \]
\[ |a_{tot}| = \sqrt{a_c^2 + a_t^2} \]
You’re looking down at a frictionless channel anchored to a frictionless horizontal table top. Ignore air resistance. A ball is shot at high speed into the channel at $P$ and exits at $R$.

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Which of the above forces is (are) acting on the ball when it is at point $Q$?

A. 1 only.
B. 1 and 2.
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E. 1, 3, and 4.

Answer: B.

**NOTE** if you shoot something into a channel it will continue at constant speed unless something changes the speed. Here, only the direction is changing so the ball should come out at the same speed you shot it in. There is NO FORCE ACTING IN THE DIRECTION OF MOVEMENT because there is no tangential acceleration, only centripetal motion! Also, it will come out with what path? Curved or straight? —> IT WILL COME OUT IN A STRAIGHT LINE, and TANGENTIAL TO THE CURVE!!!
When you turn in a car, it feels like something is pushing you outward. But there is no actual force there. The car has centripetal acceleration acting, and your body is simply trying to continue in a straight line. Try jumping off of a whirligig sometime. You will end up tangential to your exit point.
A popular application of centripetal motion is artificial gravity…
Astronauts spending lengthy periods of time in space experience negative effects due to weightlessness, such as weakening of muscle tissue. In order to simulate gravity, how many revolutions per minute would be required to create a normal force equal in magnitude to the astronaut's weight?

\[ a_c = \frac{v^2}{r} = r\omega^2 \]

Usually we have a normal force facing outward from Earth. The weight we “feel” (AIR QUOTES) is that normal force of the Earth pushing on us. For this application of centripetal force, we can see what we’d call a normal force facing upward from the “ground”—in this case, the wall of the spaceship.
A sample of blood is placed in a centrifuge of radius 16.0 cm. The mass of a red blood cell is $3.0 \times 10^{-16}$ kg, and the magnitude of the force acting on it as it settles out of the plasma is $4.0 \times 10^{-11}$ N. At how many revolutions per second should the centrifuge be operated?

Homework question related to this lecture: the same idea. Centrifuges spin blood, applying a greater force than gravity by creating a centripetal force that acts on the fluid in the vials.
Newton postulated that Gravity also acts as a centripetal force. This provides a broad idea of the solar system: everything is orbiting the sun because of its strong gravitational force pulling on the planets. If the sun's gravity suddenly disappeared, all the planets would go off at tangents to their orbits (note: this is not possible; this much mass cannot simply disappear)!
Newton postulated that Gravity also acts as a centripetal force. This provides a broad idea of the solar system: everything is orbiting the sun because of its strong gravitational force pulling on the planets. If the sun’s gravity suddenly disappeared, all the planets would go off at tangents to their orbits (note: this is not possible; this much mass cannot simply disappear)!
In fact though, if the sun suddenly vanished the planetary bodies would actually still be pulled towards one another. This is because in fact anything with mass exerts a gravitational force. And it exerts and equal and opposite force on another body. Newton formulated that the force of gravity between any two objects is equal to the multiplication of two masses divided by the square of the distance between the centers of the two objects times a constant (note: this constant has been measured). So take the Earth and Sun: The force that pulls the Earth towards the Sun (enabling our orbit), and the force that pulls the Sun towards the Earth, are of equal magnitude. How can this be? Well, as with other problems we've done for Newton's 3rd law, the sun is much more massive, about 300,000 times more than Earth. So that makes it about 300,000 times harder to move even if you apply the same force to it. So the sun moves the planets more easily than the planets move the sun.

\[ F_G = G \frac{m_1 m_2}{r^2} \]

- \( r \) = distance between centers of two objects
- \( G = 6.67 \times 10^{-11} \) N m\(^2\) / kg\(^2\) (Gravitational Constant)
Real Gravity

\[ F_G = G \frac{m_1 m_2}{r^2} \]

\( r = \text{distance between centers of two objects} \)

\( G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \)  
(Gravitational Constant)

BUT the planets do actually move the sun a little bit! The sun doesn’t actually sit at the center of the solar system; it wobbles around. Proof of Newton’s 3rd law, applying it to gravitation.
We are all gravitationally attractive!

You’re sitting 0.5 m away from your special someone. Estimate the gravitational attraction between you.

\[ F_1 = F_2 = G \frac{m_1 \times m_2}{r^2} \]

- \( m_{\text{you}} = 80 \text{ kg} \)
- \( m_{\text{ss}} = 70 \text{ kg} \)

\( G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2 \)

(Gravitational Constant)

Note: all things are gravitationally attractive, but things with more mass contribute to a larger net gravitational force between two objects. So the more net mass in the pair, the greater the gravitational force.
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Note: who is more attractive?

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