ASTR469 Lecture 10: Photometry (parts of Ch. 10)

Chapters 8–9 are not required reading but may be of interest to those who are keen.

Assess yourself/study guide after lecture & reading (without peeking at notes)...

1. Let's say you're measuring your height with a ruler. Will your measurements have a Gaussian or Poisson distribution?

2. Let's say now you measure how many times per minute a PRT car goes by between 12 and 1pm each weekday. Will your measurements have a Gaussian or Poisson distribution?

3. Is there a maximum signal-to-noise observation you can achieve when observing a very bright star?

4. In the absence of CCD noise, what approximate S/N would be measured for a star on dark sky if it has a flux of 10 W/m^2? Assume you have a telescope with a circular primary aperture of radius 0.5 m.

1 Photometry

Generically, photometry refers to quantifying the amount of light we receive.

In the past we've spoken a lot about the photons coming out of astronomical objects, and the flux incident on our telescope reflector surfaces. Now it's time to discuss actually using the photons to make something measurable. This will involve a practical discussion of the statistics of collecting photons in the presence of noise, which will be applicable to all wavelength regimes.

We'll also discuss the most common technology used to measure photons specifically in higher-energy bands: Charge-Coupled Devices (CCDs). Ultimately, these CCDs (and other measurement tools we'll discuss at other wavelength regimes) help us characterize the flux that is incident from a star onto our detector.

First, how can we tell if a source is "detected" above the noise level of our device, the sky, and other noise sources? The signal-to-noise ratio (S/N) is the important quantity for determining whether a source is detected or not. It is the ratio of the "signal", or total light collected from the source, to the "noise" or the background level (σ). While the signal is relatively easy to understand, noise takes a bit more to comprehend. We will discuss both then wrap it together to understand S/N.

2 Signal

Let's say we have a source flux F in units of $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$ (i.e. Watts per hertz per square meter). We integrate for long enough, and the signal strength is just $F \times t$, where t is the integration time. So essentially we're counting the total signal collected as the energy per square meter of the detector, at a given frequency.

3 Noise

When we're taking a measurement of anything, we don't get the perfect measurement every time (due to issues with our tools, with our estimation, even with natural instabilities in the measured object itself). Instead, if we took some large measurements of anything, there is always an error associated with a measurement that is characterized by the "noise sources" causing the variations.

3.1 Gaussian vs. Poisson Statistics

Gaussians:
Most things you measure vary with a "Gaussian" distribution (also called a "Normal distribution"). The Gaussian distribution is symmetric, and occurs specifically when you have an equal chance at measuring too high or too low. This is the traditional bell-shaped curve that applies to many situations, and is given by the probability density of obtaining a mea-

surement value of x:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, exp\Big(- \frac{(x-\mu)^2}{2\sigma^2}\Big) \tag{1}$$

The Gaussian distribution is characterized by the actual most likely signal (μ), which is the mean of the distribution, and the breadth of the distribution (σ). Mathematically, σ is the standard deviation of the measurements, but in practical terms it is also used as the uncertainty in your measurement, which gives the size of the error bar you would use. See lecture slides for reference.

Gaussian uncertainties are sometimes called "white noise." Imagine measuring some quantity. It is most likely that you measure exactly correctly (at μ), with 68% of your measurements falling within $1\,\sigma$ of the true value, 95.8% within $2\,\sigma$, and 99.8% within $3\,\sigma$.

Poissonians:
The Poisson distribution applies to situations where you're counting discrete events; the event either happens or it doesn't. Events are quantized.

I like to think of it as "balls being thrown at a person." Balls are being thrown at me at a constant rate; over some period of time, I usually catch three. Most of the time I can catch three, but sometimes I only catch two or one, and somewhat more rarely I happen to catch four in that same time frame. Similarly, you might count the number of emails you receive in one day. In astronomy, the discrete events are from photons.

In a Poisson distribution, the uncertainty σ is the square root of the number of events. If there are 100 photons counted, 68% of the tie the "true" number of photons will be between $\sqrt{100} = 10$, so between 90 and 110.

4 Noise in astronomy

So generically, if you have some signal S contributing, you will get an accompanying noise $\sqrt{S}$! Let's consider how this works generically in all astronomy.

4.1 Source noise

The source itself of course contributes to the noise in an image. This noise source is strictly Poissonian, but can be assumed to be Gaussian:

$$\sigma_{\text{source}} = \sqrt{F_{\text{source}} \times t} \tag{2}$$

4.2 Background/Sky noise

Unresolved sources in the background contribute to the overall noise level. Just like the source noise, this is Gaussian for large enough counts:

$$\sigma_{\text{bg}} = \sqrt{F_{\text{bg}} \times t} \tag{3}$$

4.3 Adding the Noises and Computing S/N

These noises combine by adding in quadrature. **SEPARATE SOURCES OF NOISE NEVER ADD LINEARLY!** If we're targeting an object, then based on the above two noise sources, we get a total noise in our star's measurement of

$$\sigma_{total} = \sqrt{\sigma^2_{source} + \sigma^2_{bg} + ...} \tag{4}$$

(where ... represents any other noise source; later we will see that our detector introduces some noise).

Putting it all together, for a star with flux F_{star} and considering only those two noise sources we'd get a S/N ratio of

$$\mathrm{S/N} = \frac{F_{\mathrm{star}} \times t}{\sqrt{\sigma^2_{star} + \sigma^2_{\mathrm{bg}}}} \tag{5}$$

or

$$\mathrm{S/N} = \frac{F_{\mathrm{star}} \times t}{\sqrt{(F_{star} + F_{bg})t}} \tag{6}$$

We frequently use multiples of the noise σ to define the S/N, such that $N \times \sigma = \mathrm{S/N}$. So a 3σ detection has three times the signal compared to the noise. We usually use 3σ as the minimum standard for a detection, whereas 5σ is more secure.

The concept of σ is useful in how it relates to Gaussian statistics. A 1σ detection has a 68.2% chance of being real for a given pixel, 95.6% for 2σ and 99.8% for 3σ. Since there are usually thousands of pixels, 3σ is not usually good enough.

4.4 How Can I Raise my S/N in an Observation?

There's one cool thing to notice about the previous equation. If you move the t's around, you see that:

$$\mathrm{S/N} = \frac{F_{\mathrm{star}}}{\sqrt{(F_{star} + F_{bg})}} \times \frac{t}{\sqrt{t}} = \frac{F_{\mathrm{star}}}{\sqrt{(F_{star} + F_{bg})}} \times \sqrt{t} \tag{7}$$

In other words, $\mathrm{S/N} \propto \sqrt{t}$ ... whoa!!! **The longer you integrate, the more sensitivity you get**, but it doesn't increase linearly with time (S/N increases more slowly). Important note: this is only true because of the fact that the source's signal becomes brighter with time faster than the noise does. **Integrating more with time will not help if your sky is brighter than your target, or if other non-signal noise sources multiply quickly with time. Sometimes we call this the "bright-source limit."**

5 Charge-coupled devices (CCDs)

CCDs use the photoelectric effect to measure incident light; within a CCD cell, the incoming light strips off an electron and holds it in that cell until it is read out by the user into a computer. CCDs are used from the ultraviolet through the infrared. See the lecture slides for images of CCDs and how they function.

A few important CCD concepts, as applied to astronomy, are:

- We don't detect photons directly! Instead we detect photoelectrons.
- To observe in narrow bands, you need external filters; CCDs will otherwise be broad-band light detectors. Usually these filters are pieces of glass or other material placed along the lens/mirror path to filter light. For reference, previously we've spoken about Johnson filters in the optical band.
- The number of pixels and pixel size in your CCD can also set your limiting resolution and field of view for a telescope.

CCDs are great for registering light, and much better than other photo-detectors, for two particular reasons (among others I'm sure):

- **They have high "Quantum efficiency."** This refers to how many photons are needed to produce one electron charge, and is also known as "gain". If each photon results in one stored electron, it is 100% efficient. Although not CCDs, our eyes are said to have effective QEs of 3% for rods and 10% for cones. Good CCDs are much better than this, maybe 40% at the ideal wavelength. Great CCDs can reach 95% efficiency. The quantum efficiency is wavelength-dependent! Each CCD, depending on how it is manufactured, will have a range of wavelengths where it is most sensitive and therefore has the highest efficiency.
- **CCDs are "linear" detectors.** This means that if you double the number of incident photons, you get twice the stored charge. In other words, the gain is constant irrespective of the incident photon flux. As we know, our eyes are not linear, which led to the magnitude system. However, eventually in a long exposure (or observing something too bright), CCDs will "saturate" (their electron storage capacity in a given pixel can fill), and you will get fewer stored electrons that you expect for a given incident flux.

5.1 Noise specific to CCDs

As previously noted, the noise is composed of every possible source of a signal being registered. CCDs add two in particular...

Read Noise
Read noise comes from the electronics when a CCD is "read out" - when the stored charges are turned into data.

Let r be the read-out noise per pixel (in electrons). The total read out noise is then:

$$\sigma_{read} = \sqrt{n_{pix} \times r^2} = \sqrt{n_{pix}} \times r \tag{8}$$

It may seem strange here to have an r^2 term, and it is, but that is just by convention. We could have just as easily defined it so that the squaring was not necessary.

Dark Current
CCDs build up "dark current" whether they are being exposed to light or not. Recall we discussed Planck spectra for black-body emission; therefore you know that whether or not something is really "bright," just the fact that it's not at 0 K means that it is to some degree emitting photons over broad frequency bands. Dark current is caused by thermally generated electrons that build up in the pixels of all CCDs, causing excess voltage when the CCD is read out.

The rate of dark current accumulation depends on the temperature of the CCD (lower temperatures better) but will eventually completely fill every pixel in a CCD. The rate can be reduced by cooling CCD.

$$\sigma_{Dark} = n_{pix} \times dk \times t \tag{9}$$

where dk is the rate for number of electrons per pixel per second.

Putting all noise sources in a CCD observation together,

$$\sigma_{total} = \sqrt{\sigma^2_{\text{source}} + \sigma^2_{\text{bg}} + \sigma^2_{\text{dark}} + \sigma^2_{\text{read}}} \tag{10}$$

And therefore

$$S/N = \frac{F \times t}{\sqrt{\sigma^2_{\text{source}} + \sigma^2_{\text{sky}} + \sigma^2_{\text{dark}} + \sigma^2_{\text{read}}}} \tag{11}$$

or

$$S/N = \frac{F \times t}{\sqrt{(F_{source} + F_{sky}) \times t + n_{pix} \times r^2 + n^2_{pix} \times dk^2 \times t^2}} \tag{12}$$