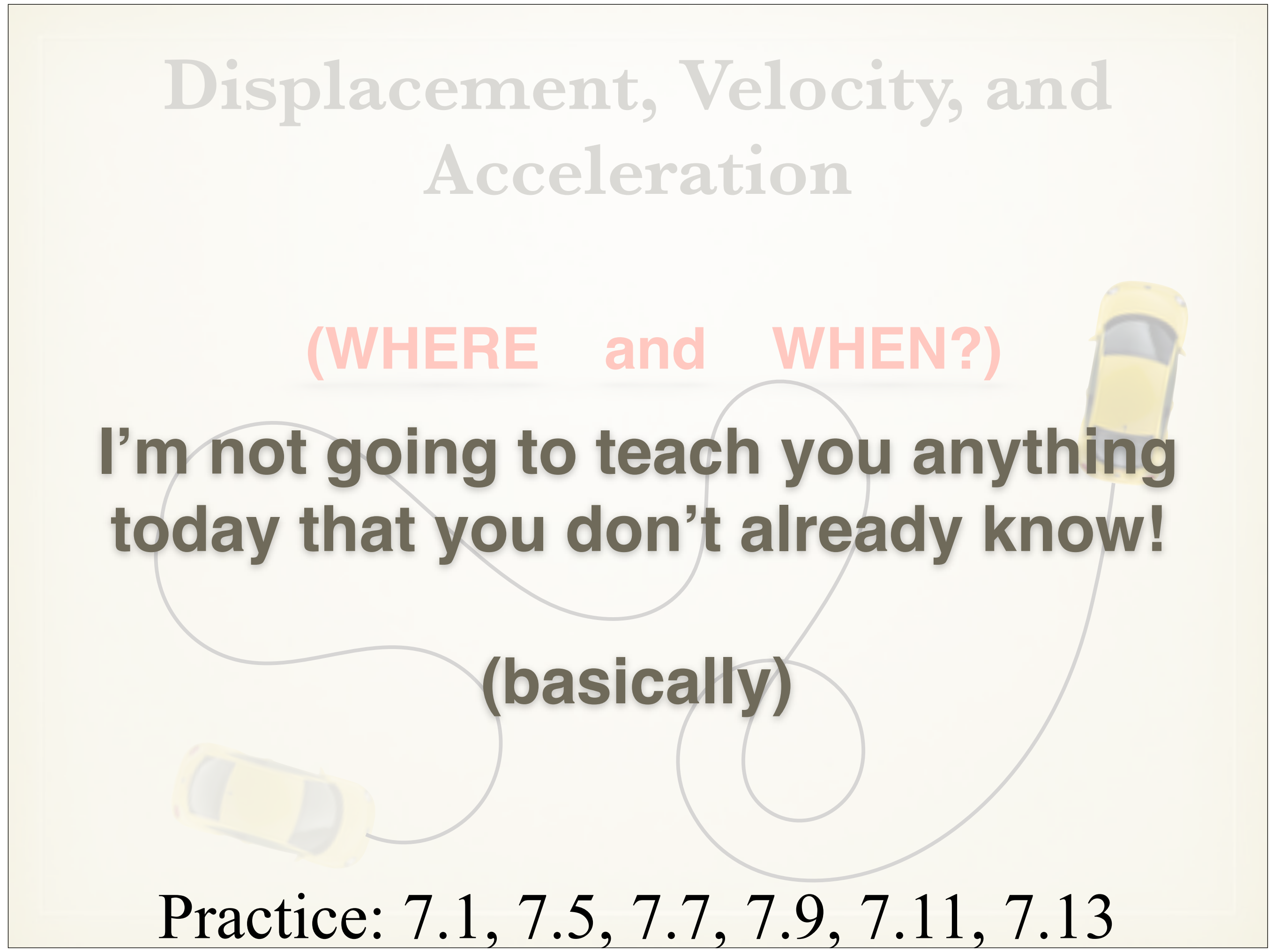
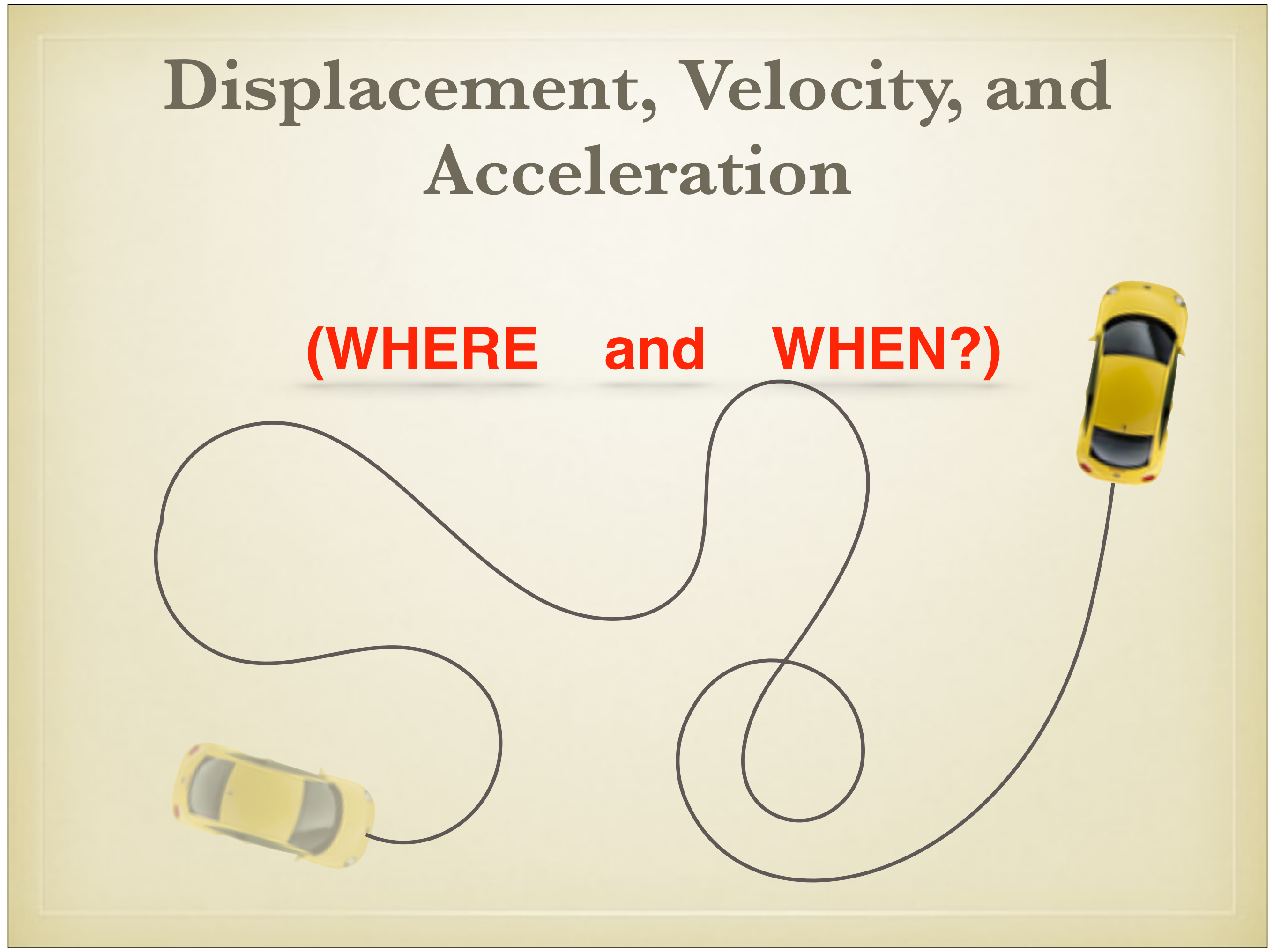


Do you guys remember this background slide? I wanted to show you where you started from to inspire you: we’ve learned a lot and most of you are doing great! Mid-semester grades have been submitted, by the way.

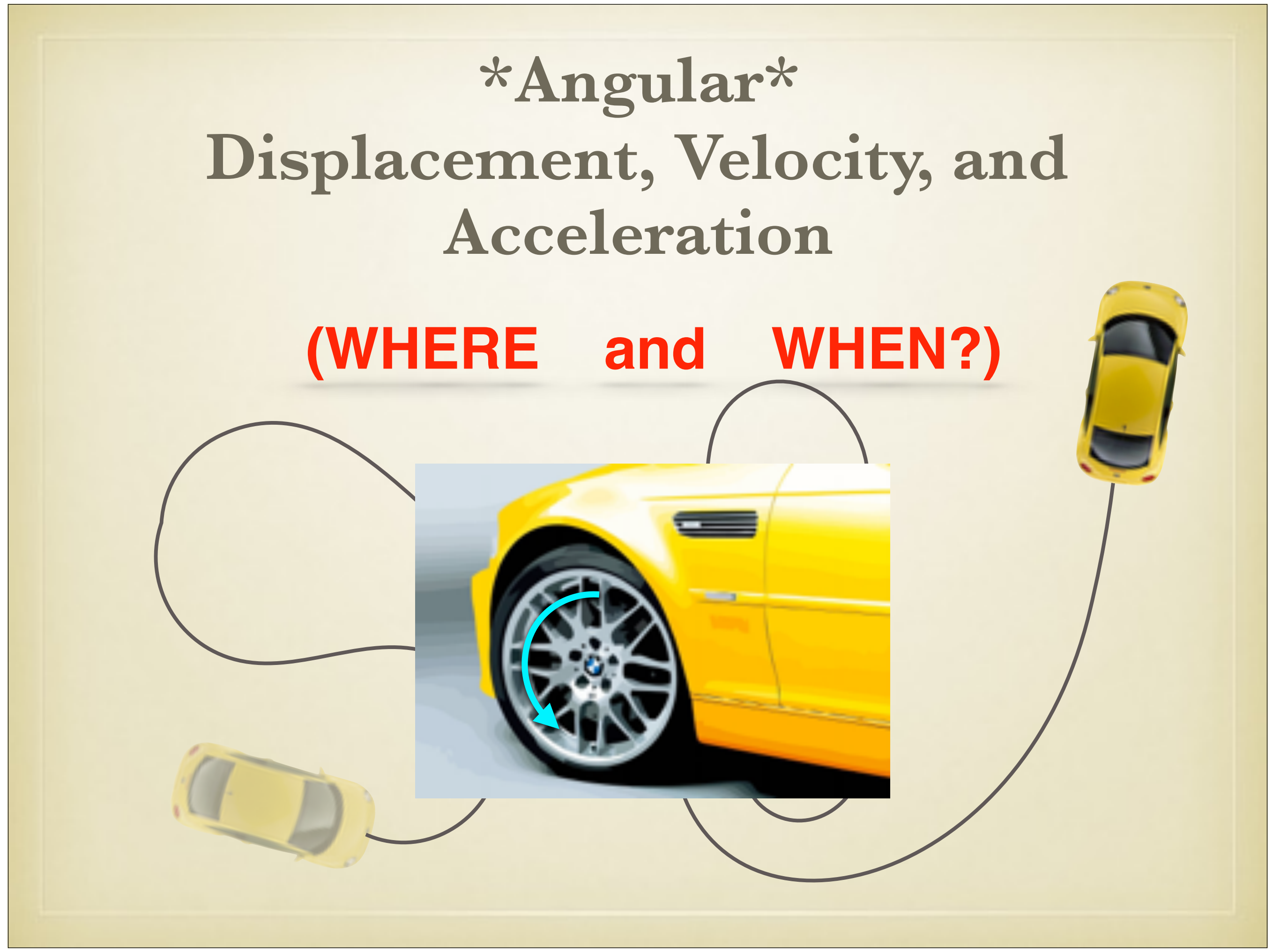
Notes

- Thanks for your feedback!
  - I will state clicker answers clearly. Remind me if I don't!
  - I will try to write bigger on light board! If things are still too small, please do me and you a favor and speak up during class! There will be zero people mad at you.



The other reason I wanted to show you this was to remind you that when we did linear motion, first we developed the equations of motion. Then we discussed the forces that cause this motion.

We're going to do something very similar now, and develop the same things for ROTATION.



Rotational motion has to do with anything spinning…
Earth
Tires
Disks

What you'll be doing…

- Unit conversion **(degrees, radians, revs)**.
- **Angular** position, velocity, acceleration.
- Convert between angular and linear quantities.

Linear motion			Angular motion
Displacement,	Δx	$\Delta\theta$	Angular displacement
Velocity,	v	ω	Angular velocity
Acceleration,	a	α	Angular acceleration

$$v = v_0 + at$$

$$\Delta x = v_0 t + \tfrac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \omega t + \tfrac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

The rotational motion equations look like weird cousins of the usual kinematics equations. We use them in very similar ways, though. All about change in position, speed/velocity, and acceleration with time.

Units of θ

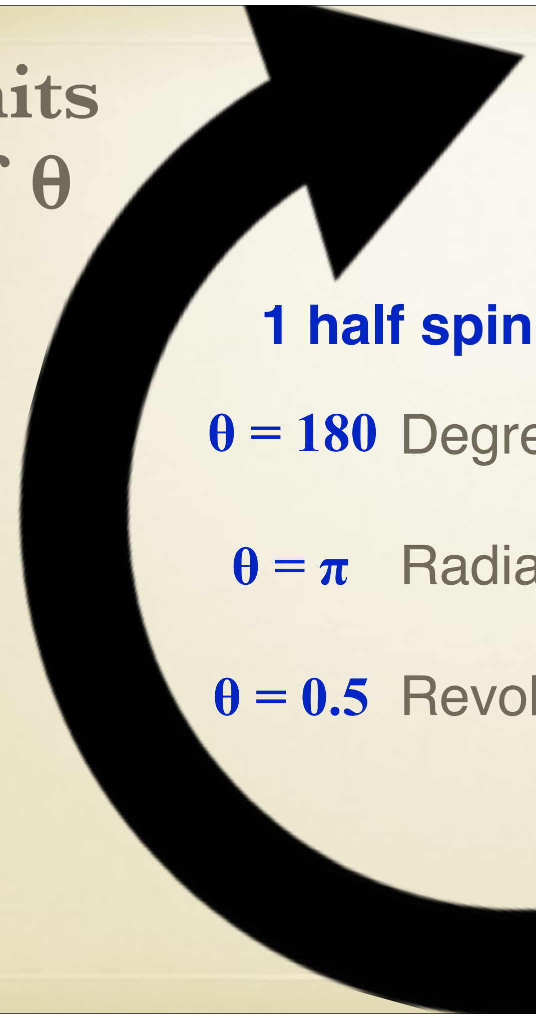
1 full spin is…

$\theta = 360$ Degrees

$\theta = 2\pi$ Radians

$\theta = 1$ Revolutions (revs)

Units of θ



1 half spin is...

$\theta = 180$ Degrees

$\theta = \pi$ Radians

$\theta = 0.5$ Revolutions (revs)

Let's convert…

Q67

Degrees to radians

$$360°\left(\frac{2\pi \ \ radians}{360°}\right) = 2\pi \ \ radians$$

Your friend wants $\pi/3$ radians of pizza. How many degrees is this?

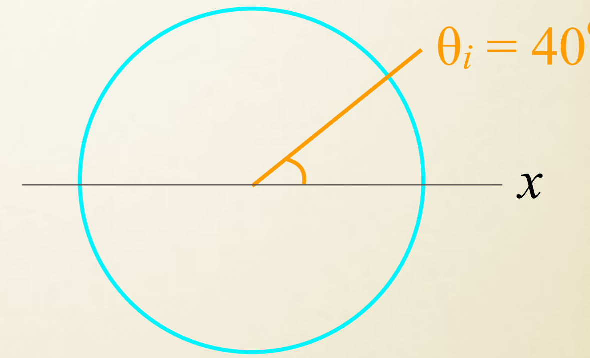
A. 30 B. 60 C. 90 D. 120 E. 240

Always multiply by 2pi/360 or pi/180 to convert degrees to radians!

Some Definitions

Angular displacement

$$\Delta\theta = \theta_f - \theta_i$$

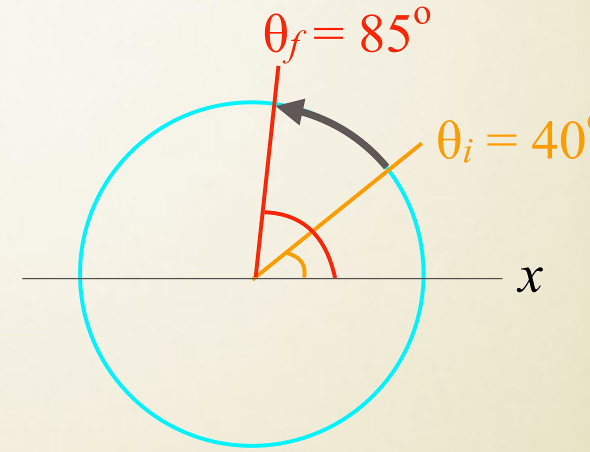


Just like we could define initial and final positions, to find linear displacement, we can do the same for **angular displacement**.

Angles usually defined upwards from x.

Some Definitions

Angular displacement



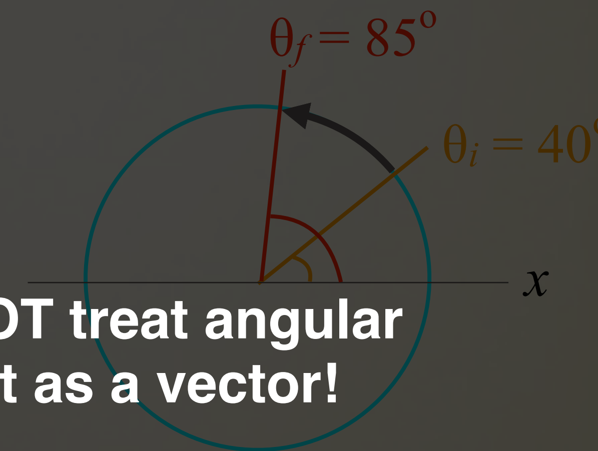
$$\Delta\theta = \theta_f - \theta_i$$

Just like we could define initial and final positions, to find linear displacement, we can do the same for **angular displacement**.

Analogous to how we derived linear displacement.

Some Definitions

Angular displacement



$$\Delta\theta = \theta_f - \theta_i$$

Note: We do NOT treat angular displacement as a vector!

Just like we could define initial and final positions, to find linear displacement, we can do the same for **angular displacement**.

Think about angles going more than one rotation: We treat each full rotation as 360 degrees, and if we go further that tacks on.

Some Definitions

Average angular velocity

Change in rotational angle over time!

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

units: rad/s

Just like we could define velocity as displacement over time, we can do the same for **angular velocity**.

Constant angular velocity: spin at some rate. If I maintain some speed at my car, my dashboard reports the rate of rotation of my engine in revs per minute (how many times does it spin in one minute)?

What is Earth's angular speed?

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

Remember there are 2π radians in a full rotation.

A. 1 rad/s
B. 2π rad/s
C. 3.14e-5 rad/s
D. 7.27e-5 rad/s

Q68

ANSWER: D. The earth spins one time (2pi radians) in 24 hours.
So 2pi rad / 24h = 7.27e-5 rad/s

Some Definitions

Average angular acceleration

Change in spin rate over time!

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\Delta\omega}{\Delta t}$$

units: rad/s^2

Just like we could define acceleration as change in velocity over time, we can do the same for **angular acceleration**.

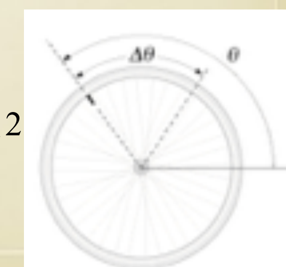
CHANGE IN SPIN RATE OVER TIME.

Constant angular acceleration: on a bike this would be using brakes (accel opposing wheel rotational velocity) or pedaling your pedals (acceleration in same direction as rotational velocity).

Same process as kinematics: List what you know in variable form, then match with a formula

Linear	Rotational
Δx	$\Delta\theta$
$\bar{v} = \dfrac{\Delta x}{\Delta t}$	$\bar{\omega} = \dfrac{\Delta\theta}{\Delta t}$
$\bar{a} = \dfrac{\Delta v}{\Delta t}$	$\bar{\alpha} = \dfrac{\Delta\omega}{\Delta t}$
For constant a:	For constant α:
$v = v_o + at$	$\omega = \omega_o + \alpha t$
$\Delta x = v_o t + \frac{1}{2}at^2$	$\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a\,\Delta x$	$\omega^2 = \omega_o^2 + 2\alpha\,\Delta\theta$

Note: the line means average!



These are not exchangeable: one applies to moving in a line, the other applies to something that is ROTATING or SPINNING.
But we can use the rotational equations in similar ways for spinning problems.

Q69

BIG BEN in London and a tiny alarm clock both keep perfect time. Which minute hand has the bigger **angular velocity** ω?

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

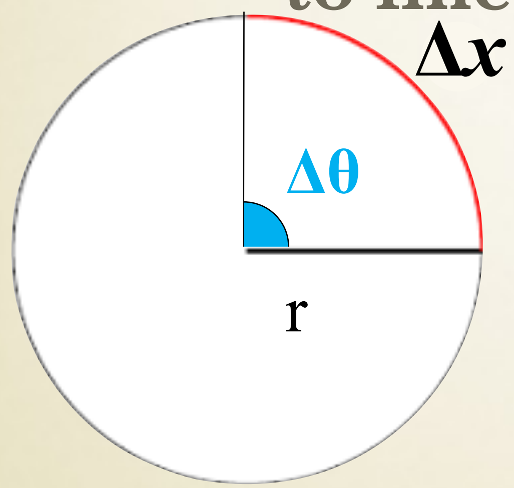
A. Big Ben
B. Little alarm clock
C. Both have the same ω

Answer: C

Converting angular motion to linear motion

For each spin of a bike tire, that translates to some distance travelled.
Think about this: if you have smaller tires, they need to spin more times to cover the same distance.

Converting angular motion to linear motion

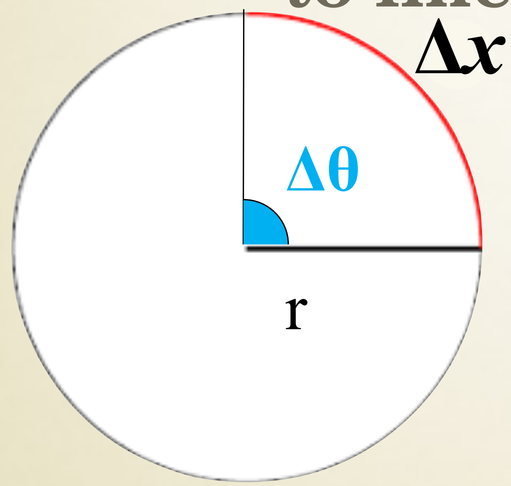


Δx = Arc length

r = Radius

$$\boldsymbol{\Delta x = r\,\Delta\theta}$$

Converting angular motion to linear motion



Δx = Arc length

r = Radius

$$\Delta x = r\,\Delta\theta$$

What is Δx for one rotation (circumference of a circle)?

A. πr
B. 2πr
C. πr2
D. 2πr2

Q70

Answer: B.

CAREFUL!

Be careful about this next step… it’s one of the more confusing things about rotational analysis

The tires on a car have a diameter of 0.5 m and are warrantied for 100,000 km. Determine the angle (in radians) through which one of these tires will rotate during the warranty period.

$$\Delta\theta = \frac{\Delta x}{r}$$

How many revolutions of the tire are equivalent to our answer?

Answer: 4x10^8. But 4x10 WHAT?

You'll notice that this seems unitless. IT'S NOT. IF YOU ARE USING THIS EQUATION, IT COMES OUT IN RADIANS.

This is the definition of a radian. Radians are ratios of the arc length described by an angle to the radius of the arc. Revolutions: did on light board. Remember there are 2pi radians per rotation. And one revolution of the wheel per rotation! So convert by multiplying your radians answer by 1rev/(2pi rad).

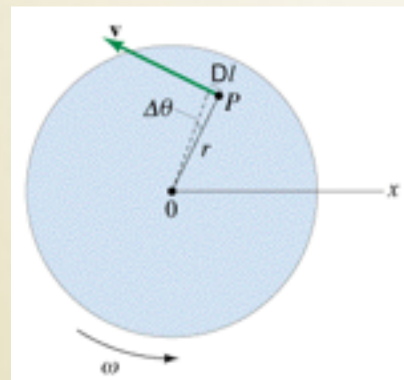
Converting angular motion to linear motion

If you're sitting near the middle, for each spin are you moving very far? Over how much time? What if you're sitting on the edge? How far over how much time?

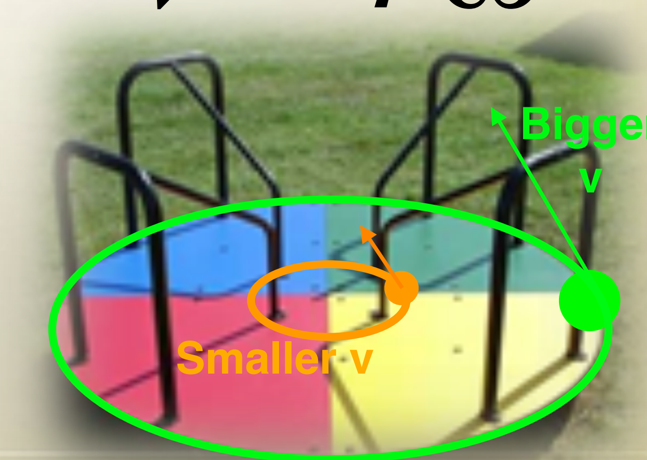
Converting angular motion to linear motion

Linear to angular velocity

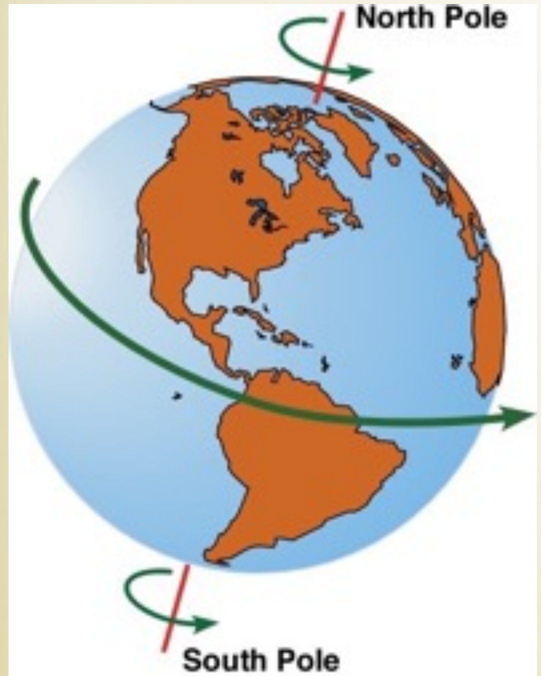
v is linear speed of point *P*



$$v = r\omega$$



I wanted to mention this because at each point on this rotation, you have an angular speed, your number of turns per amount of time, but if your radius is larger, you're covering more ground (more Delta x) in the same amount of time.



If you're standing at the equator, what is your linear speed with respect to the Earth's center? (Earth's radius is ~ 6400 km.)

$$v = r\omega$$

Converting angular motion to linear motion

$$\Delta x = r\,\Delta\theta$$

$$v = r\,\omega$$

$$a = r\,\alpha$$

If you noticed a pattern here, you're right. You can convert angular and linear motion values by multiplying the ANGULAR VALUE by the radius of the circle.

Kids' tricycle

I put stickers on the bottom of the front and back wheels of different sizes. As I roll this tricycle (without slipping), the stickers complete a circle (360 degrees) at:

A. The same time
B. Different times
C. Depends on the speed of the bike

$$v = r\,\omega$$

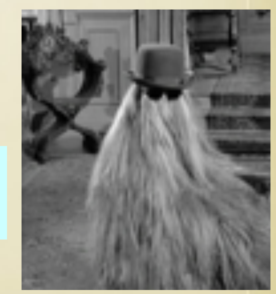
Answer: B.

Same process as kinematics: List what you know in variable form, then match with a formula

Linear	Rotational	
Δx	$\Delta\theta$	$\Delta x = r\Delta\theta$
$\bar{v} = \dfrac{\Delta x}{\Delta t}$	$\bar{\omega} = \dfrac{\Delta\theta}{\Delta t}$	$v = r\omega$
$\bar{a} = \dfrac{\Delta v}{\Delta t}$	$\bar{\alpha} = \dfrac{\Delta\omega}{\Delta t}$	$a = r\alpha$

Note the line means average!

For constant a:	For constant α:
$v = v_o + at$	$\omega = \omega_o + \alpha t$
$\Delta x = v_o t + \frac{1}{2}at^2$	$\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$
$v^2 = v_o^2 + 2a\,\Delta x$	$\omega^2 = \omega^2{}_o + 2\alpha\,\Delta\theta$



I showed you this before but wanted to solidify again. Now we have a new set of equations equations, and some conversions.

To throw a curve ball, a pitcher gives the ball an initial angular speed of 36.0 rad/s. When the catcher gloves the ball 0.595 s later, its angular speed has decreased (due to air resistance) to 34.2 rad/s.

(a) What is the ball's angular acceleration, assuming it to be constant?
(b) How many revolutions does the ball make before being caught?

$$\Delta\theta \qquad \omega = \omega_o + \alpha t$$

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \qquad \Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} \qquad \omega^2 = \omega_o^2 + 2\alpha\,\Delta\theta$$

Now we just have to practice interpreting what the problem is asking us to do! Careful when you're writing your variables and considering what the problem is asking for!

The first question: asking for average acceleration.

The second question: How many times does the ball spin? This is asking for DELTA THETA, converted from radians to revolutions! The total rotations/revolutions of the ball. You'll have to use the second or third rotational kinematic equation to calculate this.

Have a great spring break!