ASTR469 Lectures 7–8: Coordinates (Ch. 1)

Assess yourself/study guide after lecture & reading (without peeking at notes)...

1. How do we measure locations on Earth? What coordinate system do we use and where are the zero-points?

2. Check your grasp on terminology: what are the differences between zenith, altitude, elevation, and azimuth angles?

3. Right ascension and declination are convenient because they relate to finding stars based on Earth's position. Consider: for what objects might it be (conceptually/scientifically) convenient to use Galactic coordinates?

4. If you know a bit of code, try to write a script that will convert between Galactic and Equatorial coordinates. Compare your results to the convenient online converter tool: https://heasarc.gsfc.nasa.gov/cgi-bin/Tools/convcoord/convcoord.pl

5. What is the zenith angle of the North Celestial Cap?

6. In a span of 24 hours, how many hours is a galaxy at $\delta = +75°$ visible from Morgantown?

7. What is the largest and smallest zenith angle that a galaxy at $\delta = +85°$ will have in the sky when observing from Morgantown?

1 Using Spherical Coordinates for Sky Positions

In our first class, we discussed spherical coordinates θ, ϕ, and r. We've also said before that all sky positions are projected on to the "celestial sphere"— the imaginary surface on which we observe celestial objects. It is helpful to think of this being a real physical sphere that rotates approximately once per day (though note there is a difference between Sidereal and Solar rate, which will be explored later). Due to this apparent projection, we don't usually know/care about coordinate r. We can therefore set $r = 1$ and express directions only using the two angles. This makes life considerably easier! One important consequence of this is that measurements on the sky are in *angles*, rather than physical distances.

To define a coordinate system, we therefore need just five things:
1) A longitude coordinate
2) A latitude coordinate
3) A starting point for longitude
4) A starting point for latitude (called the fundamental plane)
5) The origin
(The poles are necessarily defined by the above choices.)

Astronomers use many different coordinate systems, depending on what is most convenient, but all of these systems use spherical coordinates. It is important to understand these systems, why they exist, and how we can convert between them.

We'll go through these five characteristics of each coordinate system.

2 Coordinate Systems

2.1 Defining Locations on the Earth

Practical usage: Defining where we are standing on the globe (Fig. 1).

On the Earth we use:
1) Longitude to specify the E-W direction. Ranges from 180° East to 180° West, uses λ.
2) Latitude to specify the N-S direction, ϕ. Ranges from 90° north ($+90^\circ$) to 90° south (-90°).
3) The longitude of Greenwich, England is at zero degrees longitude.
4) The equator is at zero degrees latitude.
5) The center of the Earth as the origin.

While you all probably know all this, we will soon see that all coordinate systems are similar to this one that you are familiar with. One difference is that we are on the surface of the Earth, whereas we imagine that we are inside the sphere of the sky (the "celestial sphere") looking out. The effect of this change in perspective is that the direction of increasing longitude (East/West) is flipped. When looking at an astronomical image, you will often hear leftwards is referred to as "East" and rightwards referred to as "West", which is the

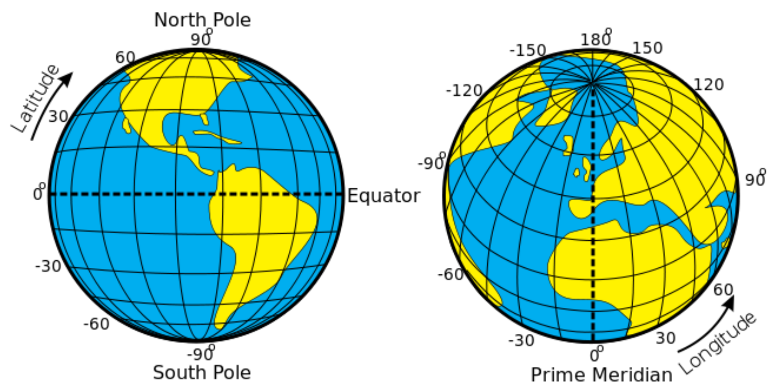

Figure 1: Coordinates used to define a location on Earth.

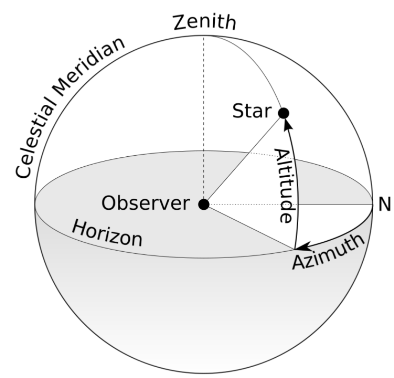

Figure 2: Local coordinates. Note the horizon, zenith, and celestial meridian. The star shown has an elevation measured from the horizon toward zenith, and a zenith angle measured from the zenith down toward the horizon. Notice also that increasing longitude (azimuth) goes in the opposite direction from that on earth.

opposite of how we define the cardinal directions on Earth.

3 Defining Locations on the Sky

3.1 Local (Horizon) Coordinates

Practical usage: Defining how far up and what direction you need to stand to see something (Fig. 2).

Perhaps the easiest celestial system to visualize is is the Local Coordinate system. This system is aligned with your local horizon, so it is tied to you the observer. It has:
1) Azimuth, Az, ranges from 0° to 360° and is measured from North toward East
2) Elevation or altitude measured from the horizon (0°) to your zenith straight over head ($+90^\circ$). Although you can't see it, at -90° is our nadir. Zenith angle, $\mathrm{ZA} = 90 - \mathrm{El}$ is an alternative angle often used for telescopes that only observe close to the Zenith.
3) North is Az$= 0^\circ$.

4) Your horizon is at El = 0°.
5) The origin is your location as observer.

While the horizon system is intuitive, because it is unique to each observer, it is not as generally useful as one would hope. We therefore have numerous other systems that are not tied to a specific observer. Horizon coordinates are useful when we are trying to determine when an object will rise and set.

3.2 Equatorial coordinates (RA/Dec)

Practical usage: This coordinate system is fixed on the the celestial sphere, so you can work out where an object should be if you know the time and date (3).

Of all these systems, the Equatorial system is most often used because it corresponds most closely with that needed to perform observations. This system is aligned with the orientation of the Earth. As we will see later, the fact that the Earth's orbit actually precesses does cause an inconvenience with this system in the longer-term. The system uses:
1) Right ascension, R.A., for longitude, which takes the symbol α. RA ranges from 0 to 360° and there are no negative values.
2) Declination, Dec., which takes the symbol δ, is used for latitude. Dec. ranges from -90° to $+90^\circ$.
3) The origin of right ascension is the location of the Sun on the vernal equinox (March 21).
4) Zero degrees declination is the celestial equator, the projection of the Earth's equator onto the celestial sphere.
5) The center of the Earth is the origin.

Other tidbits:
1) NCP and SCP stand for the north and south celestial poles, i.e. $\delta = +90^\circ$ and $\delta = -90^\circ$
2) The altitude of the NCP (or SCP) is your latitude on Earth
3) The Ecliptic is the Sun's annual path across the celestial sphere. Since the apparent Solar motion is caused by the Earth's revolution around the Sun, and the Earths spin axis is tilted by 23.5° to its orbital axis, the ecliptic is tilted by 23.5° with respect to the celestial equator.
4) The Vernal equinox is where the Ecliptic crosses the celestial equator, so it's not such a crazy place to define $\alpha = 0^\circ$.

3.2.1 Notation in the Equatorial System

R.A. can be expressed in degrees (or radians), but you'll usually see it written in hours, minutes and seconds of time, where 24hours = 360° or 1hour = 15°. We will see why we add this complication a bit later. So, for example, 20h34m45s = $(20+34/60+45/3600)\times 15 = 308.7^\circ$. Similarly, declination can be expressed in degrees or radians, but you'll usually see it written in degrees, minutes and seconds. So for example 09°45m34s = $9+45/60+34/3600 = 9.8^\circ$.

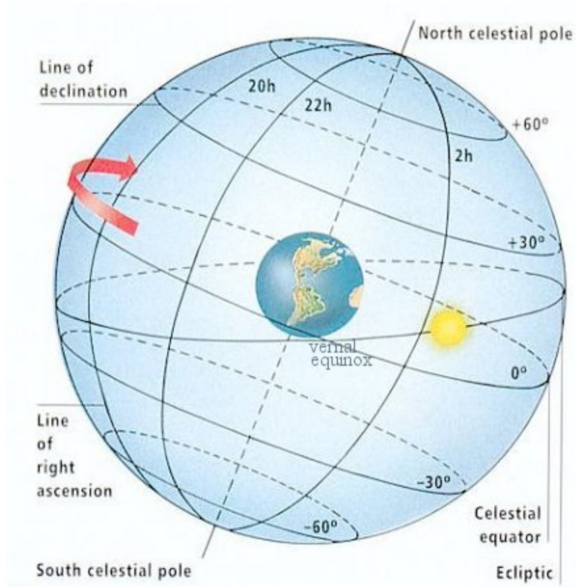

Figure 3: Equatorial coordinates. Note the north and south celestial poles, the celestial equator, and the ecliptic. The Vernal equinox is where the ecliptic and celestial equator intersect. *This system is the exact analog of what we use on the surface of the Earth.*

Although the 24 hours is strange, it is of course common to use minutes and seconds to distinguish fractions of an angle. In astronomy, we use "arcminutes" (symbol $'$) and "arcseconds (symbol $''$) to illustrate that these are angles on a curved surface. One important and strange caveat is that arcseconds of R.A. are not equal to arcseconds of Dec.! If the unit "arcsecond" is used without a coordinate attached, assume what is meant is that corresponding to the declination unit, i.e. 1/3600th of a degree.

3.2.2 The Sun in Equatorial Coordinates

Throughout its yearly motion across the sky, the Sun of course passes through a range of coordinates. It begins on March 21 at $\alpha = 0$h, and then advances approximately two hours per month over the next 12 months. Each week therefore adds about 30m to the R.A. of the Sun. In Declination, the Sun is at $\delta = 0\,^\circ$ on March 21, then advances to $\delta = +23.5\,^\circ$ on the summer solstice (June 21), back to $\delta = 0\,^\circ$ on the Fall equinox (September 21), and to $\delta = -23.5\,^\circ$ on the winter solstice (December 21).

3.2.3 Epochs

The Equatorial system is really useful, but has one strange quirk. Because the Earth's axis is not stable but rather precesses and nutates, the location of the NCP, SCP, and Vernal Equinox shifts slightly. These changes are small, but important enough that we specify an "epoch" by convention every 50 years. The most common epoch currently is the Julian

(J2000), and previously the Besselian (B1950) was used. As an example, a source with J2000 coordinates R.A. = 09 h 45 m 30 s and Dec. = -15 deg 31 m 20 s has equivalent B1950 coordinates of R.A. = 09 h 43 m 06 s and Dec. = -15 deg 17 m 28 s. While you can specify coordinates for observation in any epoch you like, it is very important to know which system you are using! Failure to keep track of this has led to untold wasted telescope hours :(.

3.2.4 A Few More Words about Declination

The declination measures angular distance from the NCP or SCP. The motion of stars across the sky is determined by your latitude on Earth, ϕ. For the northern hemisphere:
Sources with $\delta > 90^\circ - \phi$ are always visible. We call such stars "circumpolar". These same stars are never visible in south for same observing latitude.
Sources with $\delta < \phi - 90^\circ$ are never visible (circumpolar in the south for same latitude).
Low dec. sources do not rise far above the horizon.

For the southern hemisphere:
Sources with $\delta < \phi - 90^\circ$ are circumpolar.
Sources with $\delta > 90 - \phi$ are never visible.

In the northern hemisphere on Earth, *all* sources appear to circle around the NCP throughout the day/night. In the southern hemisphere, the same is true for the SCP. The full motion of every object takes just under 24 hours: actually, stars take 23 hours, 56 minutes, and 4.1 seconds to return to the same local coordinate in the sky. That duration is called the "sidereal day," and is shorter than a Solar day due to the added complexity of Earth's orbital motion around the Sun.

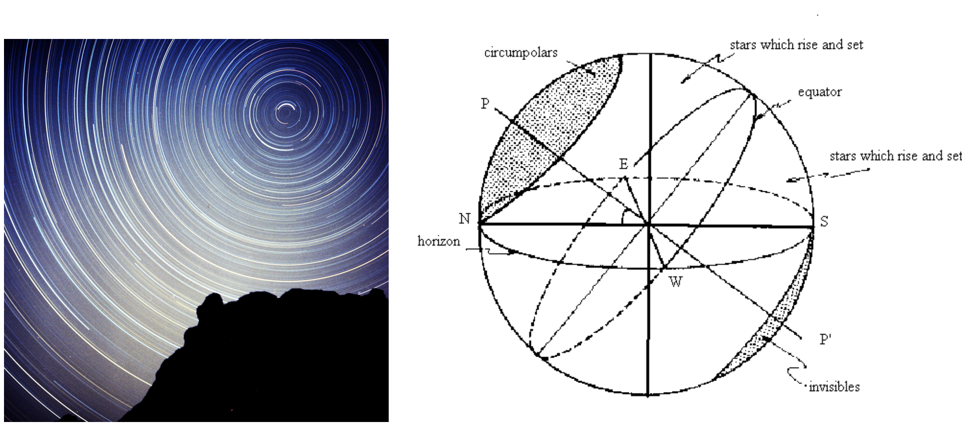

Figure 4: *Left:* A portion of the sky from a long-exposure photograph taken in the northern hemisphere. Shown are star tracks near the circumpolar region. These stars have high Dec. values. The NCP ($\delta = +90^\circ$) is at the center of the nested arcs. Each star track is roughly 1/3 of a circle, indicating that this exposure was roughly 8 hours long. *Right:* The celestial sphere showing the paths the stars take during one 24 hour period for one particular latitude on Earth. Note the circumpolar and "never rise" regions.

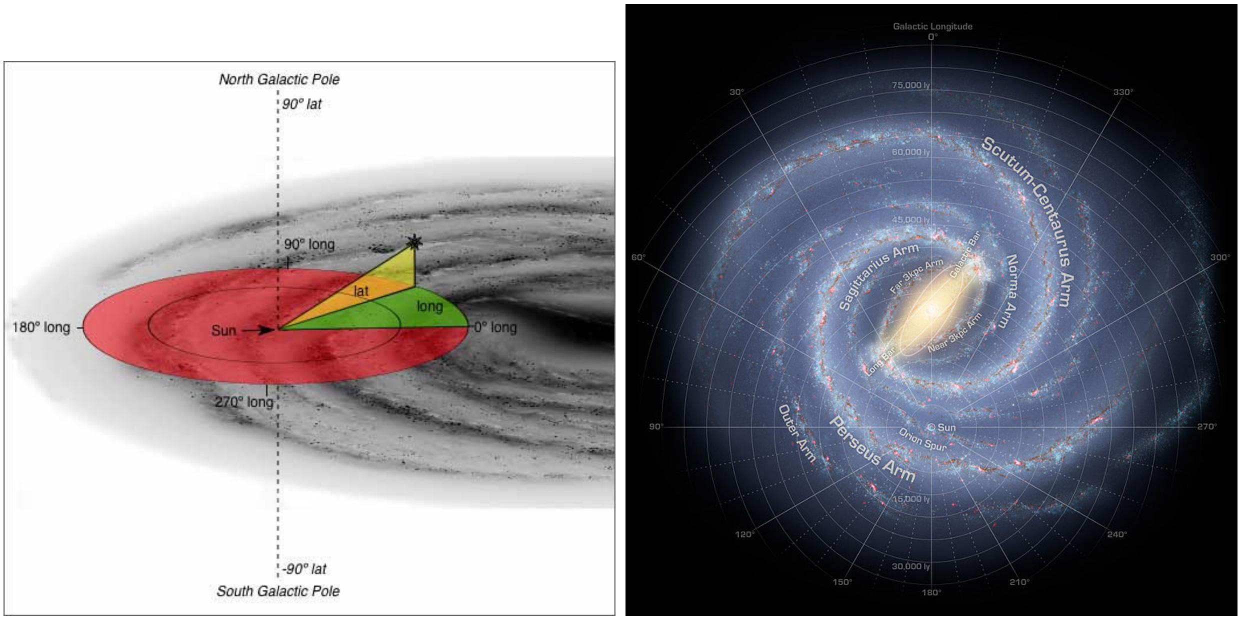

Figure 5: Galactic coordinates. The top-left panel shows how the angles are measured. Note the direction of the north and south Galactic poles. The top right panel shows a face-on view of what we think our Galaxy looks like, with longitude directions shown. Longitude increases counter clockwise from the Galactic center. The bottom panel shows in infrared view of the Galaxy from our perspective on Earth. The center of the figure is the Galactic center $(\ell, b) = (0\,^\circ, 0\,^\circ)$. The bright band in the middle is the mid-plane ($b = 0\,^\circ$). Galactic longitude increases from the center toward the left hand side, with both the left and right edges at $\ell = 180\,^\circ$. Galactic latitude is $b = +90\,^\circ$ at the top and $b = -90\,^\circ$ at the bottom.

3.3 Galactic Coordinates

Practical usage: Tells you what position an object appears to be in reference to the center of our Galaxy (Fig. 5).

Observations of objects in the Milky Way often make use of Galactic coordinates. This system is useful for specifying where objects are in relation to the rest of the Galaxy:
1) Galactic longitude, ℓ. Ranges from $0\,^\circ$ to $360\,^\circ$.
2) Galactic latitude, b. Ranges from $+90\,^\circ$ at the Galactic north pole to $-90\,^\circ$ at the Galactic south pole.
3) The Galactic center is at $\ell = 0\,^\circ$.
4) The Galactic mid-plane is at $b = 0\,^\circ$.
5) The Sun is the origin.

Stars in our Galaxy are found in higher densities toward the mid-plane and toward the Galactic center. Therefore, by knowing the Galactic coordinates of an object, we can possibly infer something about its environment. The caveat here is that we often do not know the distance to the object, which can make such inferences less accurate.

4 Converting Between Coordinate Systems

With so many systems in use, it is essential to be able to convert between them. To do so accurately, one should use spherical trigonometry.

4.1 Local and Equatorial

It is often important to convert between local and equatorial systems, to determine for example when a source rises and sets. To do so, use:

$$\sin \mathrm{Az} = -\frac{\sin \mathrm{HA} \cos \delta}{\cos \mathrm{El}} \tag{1}$$

$$\sin \mathrm{El} = \sin \delta \sin \phi + \cos \delta \cos \phi \cos \mathrm{HA}\,, \tag{2}$$

where ϕ is your latitude on Earth.

4.2 Galactic and Equatorial

The Galactic plane is tilted by about $60\,^\circ$ from the CE and intersects the CE at $\ell \simeq 33\,^\circ$.

To convert between Galactic and equatorial coordinates:

$$\sin b = \cos \delta \cos \delta_g \cos(\alpha - \alpha_g) + \sin \delta \sin \delta_g \tag{3}$$

$$\tan(\ell - \ell_g) = \frac{\tan \delta \cos \delta_g - \cos(\alpha - \alpha_g) \sin \delta_g}{\sin(\alpha - \alpha_g)} \tag{4}$$

$$\sin \delta = \cos b \cos \delta_g \sin(\ell - \ell_g) + \sin b \sin \delta_g \tag{5}$$

$$\tan(\alpha - \alpha_g) = \frac{\cos(\ell - \ell_g)}{\tan b \cos \delta_g - \sin \delta_g \sin(\ell - \ell_g)}\,, \tag{6}$$

where $\alpha_g = 192.25\,^\circ$, $\delta_g = 27.4\,^\circ$, and $\ell_g = 33\,^\circ$ define the conversion from B1950 equatorial coordinates to the Galactic frame, which was is defined at the B1950 epoch.

There are tools people have developed online (e.g. Python libraries) to help you with conversion. For one-off conversions, however, it is easy to use an online converter:
https://heasarc.gsfc.nasa.gov/cgi-bin/Tools/convcoord/convcoord.pl

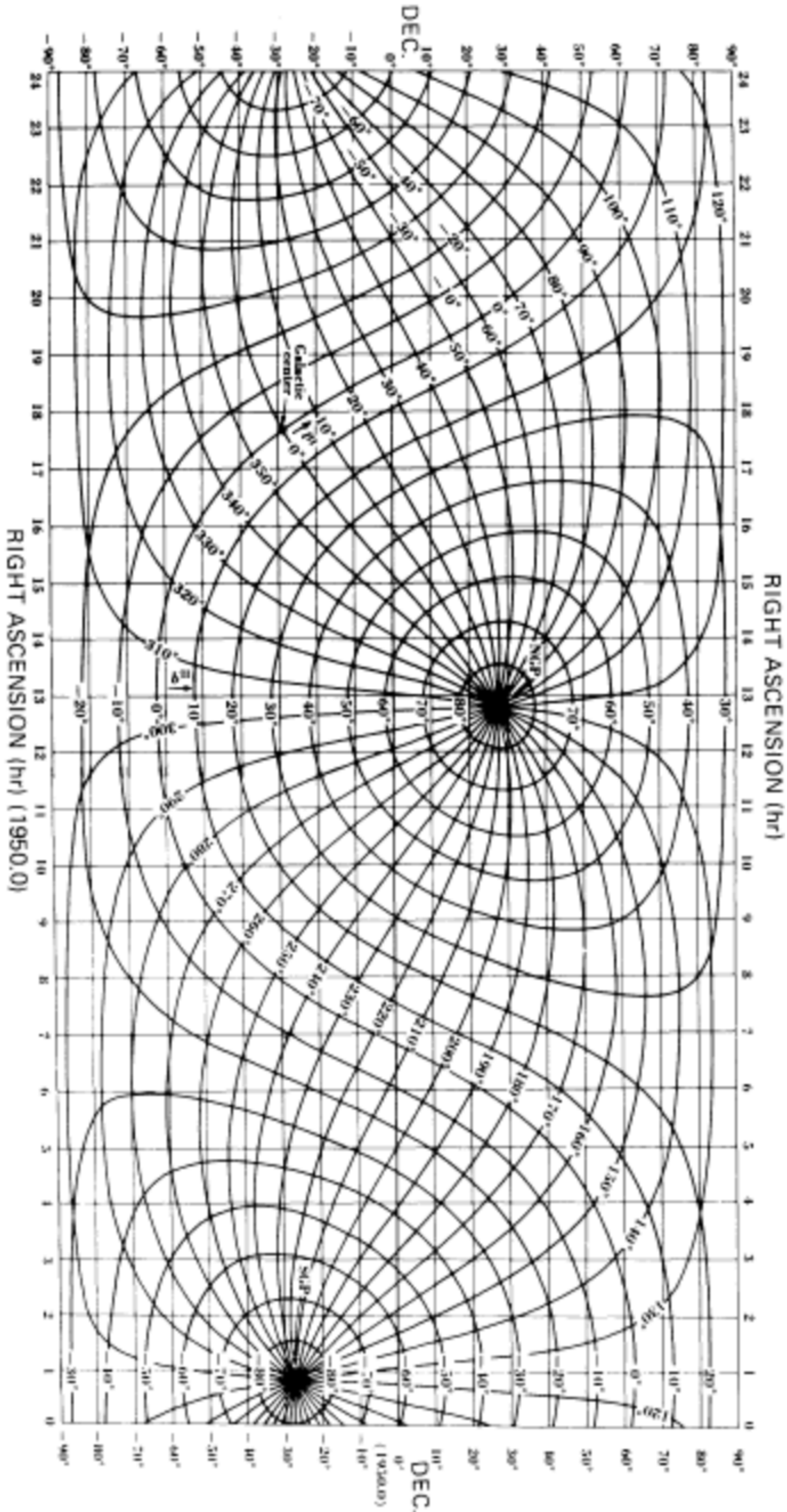

Figure 6: A map showing the conversion between Galactic coordinates and B1950 Equatorial coordinates.