Today: rotational versions of familiar things...

- Mass/inertia (Moment of inertia)
- Kinetic energy (Rotational KE)
- Momentum (Angular momentum)
- Impulse ($\tau dt = \Delta$ angular momentum)
- Conservation of momentum (of angular momentum)

...and conservation of energy

Extra Practice: 8.43, 8.49, 8.51, 8.55, 8.65

Today’s Menu

- Moment of inertia, $I$ (kinda like *mass*)
- Rotational momentum, $L$ (kinda like $p$)
- Rotational kinetic energy, $KE_r$ (kinda like $KE_i$)

Reminder: Torque

$$\tau = rF_\perp$$

**moment of inertia ($I$)**

Relates net torque ($\Sigma\tau$) to angular acceleration ($\alpha$).

- Remember Newton's second law: $\Sigma F = ma$
- Rotational version: $\Sigma\tau = I\alpha$

THINGS KEEP MOVING at same $v$

THINGS KEEP SPINNING at same $\omega$

UNLESS YOU APPLY A FORCE! UNLESS YOU APPLY A TORQUE!
Moment of Inertia

Think of it as "takes more effort to rotate"

If the mass and outer R are the same, which one of these I is smallest?

Race of the Geometrical Shapes

Which shape will win the race?

Note: since all of these shapes have the same mass, they all have the same force that will act to get them rolling down the incline (equal torque).

All have the same mass and radius

\[ \tau = rF \]

\[ \Sigma \tau = I \alpha \]

They all have the same torque. Thus, smaller I means a larger angular acceleration, which is also a larger linear acceleration.
Rotational Kinetic Energy

\[ KE_r = \frac{1}{2} I \omega^2 \]

It requires extra energy to rotate things!

A solid 1.20 kg disk \((I = 0.5mr^2)\) with a radius of 10.0 cm rolls without slipping. The linear speed of the disk is \(v = 1.41 \text{ m/s}\).

(a) Find the translational (linear) kinetic energy.
(b) Find the rotational kinetic energy.
(c) Find the total kinetic energy.

Conservation of Energy

Work done by things like friction \((\approx 0 \text{ if not slipping})\) = Sum of PE, KE\(_f\), KE\(_i\) final - Sum of PE, KE\(_f\), KE\(_i\) initial
Angular Momentum

Angular momentum
\[ L = I \omega \]
(Remember \( I = \sum m r^2 \))
Units:
kg m²/sec
[Could also be written rad kg m²/sec]

Conservation of (Angular) Momentum

Angular momentum
\[ \Sigma L_f = \Sigma L_i \]
is conserved in an isolated system!

Conservation of (Angular) Momentum

If a net torque rotates a system, the angular velocity (and the angular momentum) of that system changes.

\[ \tau_{net, system} = \sum \tau = \frac{\Delta L}{\Delta t} \]

A Windmill

In a light wind, a windmill experiences a constant torque of 255 N m.

If the windmill is initially at rest, what is its angular momentum after 2.00 s?

\[ \tau = \frac{\Delta L}{\Delta t} \]

Notice that you did not need to know the moment of inertia of the windmill to do this calculation.
### Relations

**Linear Motion**
- Mass \( m \)
- Linear velocity \( v \)
- Translational KE \( \frac{1}{2}mv^2 \)
- Linear momentum \( p = mv \)

\[
F_{net} = \sum F = \frac{\Delta p}{\Delta t}
\]

\((F=ma)\) when \( m \) is constant

**Rotational Motion**
- Moment of inertia \( I \)
- Angular velocity \( \omega \)
- Rotational KE \( \frac{1}{2}I\omega^2 \)
- Angular momentum \( L = I\omega \)

\[
\tau_{net} = \sum \tau = \frac{\Delta L}{\Delta t}
\]

**Note:** if \( I \) is constant, \( \frac{\Delta L}{\Delta t} = I\omega \rightarrow I\Delta\omega = I\Delta\omega \)

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**Extreme Fusion:**
- Star explodes!

**Gravity collapse core:**
- Core radius \( r = 2.3 \times 10^8 \) m
- Neutron star radius \( r = 10 \) km

Before star dies, it rotates at \( \omega_b = 2.4 \times 10^6 \) rad/s. What is the final spin rate of the neutron star?

**Assume solid sphere:** \( I = \frac{2}{5}mr^2 \)

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**This actually happens a lot!**

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A spinning figure skater pulls his arms in as he rotates on the ice. As he pulls his arms in, what happens to his angular momentum \( L \) and kinetic energy \( KE \)?

A. \( L \) and \( KE \) both increase.
B. \( L \) stays the same; \( KE \) increases.
C. \( L \) increases; \( KE \) stays the same.
D. \( L \) and \( KE \) both stay the same.
Consider a rod of uniform density with an axis of rotation through its center and an identical rod with the axis of rotation through one end. Which has the larger moment of inertia (more difficult to rotate)?

A: \( I_C > I_E \)  
B: \( I_C < I_E \)  
C: \( I_C = I_E \)