📡 "Back after 2-week spring break" Warm-up Questions 📡

How is emission measure defined?

It's the integral of the squared electron density along the line of sight.

Based on the emission processes we've seen so far, generally what happens to a spectrum when the medium gets optically thicker at lower frequencies?

It turns over. No emission mechanism can have its energy go to infinity at high or low frequency; self-absorption always makes the spectrum turn over as the region gets optically thick.

What kind of medium makes thermal bremsstrahlung emission?

Any region that has a thermal velocity distribution, and is in LTE.

What kind of medium makes relativistic magnetobremsstrahlung emission?

Any region with ambient magnetic fields and relativistic electrons.

What is the biggest difference in electron population property in the media in which thermal bremsstrahlung vs synchrotron emission are occurring?

The biggest differences are in the velocities. To quote ERA: "Electrostatic and magnetic bremsstrahlung are not synonymous with thermal and nonthermal radiation, respectively. For example, electrons with a relativistic Maxwellian energy distribution are in LTE and can emit thermal synchrotron radiation."

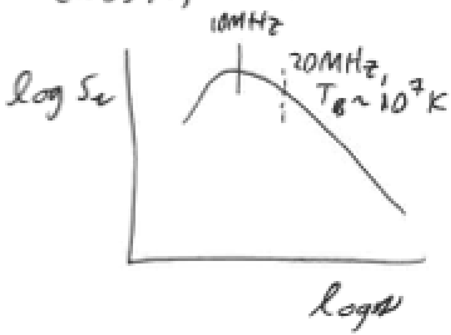
Cassiopeia A spectral shape

Cassiopeia A is a stellar remnant—a cloud of ionized gas from a star that exploded 330 years ago. Its brightness temperature at around $f = 20$ MHz is around 10^7 K. However, there is a sharp decrease in Cass A's flux density at a frequency of around 10 MHz. If this source is 3kpc distant, and the average electron density along the line of sight in the intervening medium is 0.03 cm^{-3}, is it possible that the cause of the fall off is free-free absorption by electrons along the line of sight? Hint: What defines the turn-over point for the spectra of the emission mechanisms we've discussed so far?

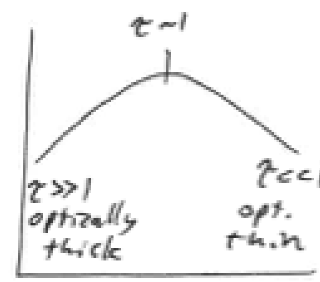
Hint 2:

Recall $T_B = T(1-e^{-\tau})$	opt thin $\tau \ll 1$, $T_B \simeq T\tau$	opt thick $\tau \gg 1$, $T_B \simeq T$	$T_B \to T$ as $\tau \to \infty$

Let's start by sketching the assumed spectrum of Cass A, which is noted to have a turn over @ 10MHz:



Is this turn-over due to free-free/thermal bremsstrahlung? The absorption turn-over happens always when $\tau \sim 1$



For TBS we derived:

$$\tau \simeq 0.0824 \left(\frac{T_e}{K}\right)^{-1.35} \left(\frac{\nu}{GHz}\right)^{-2.1} \left(\frac{EM}{pc\ cm^{-6}}\right)$$

And Cass A is 3kpc away through a medium with $\langle n_e \rangle = 0.03\,cm^{-3}$,

so $EM = \int n_e^2\, dl \simeq (0.03/cm^3)^2 \times 3000\,pc = 2.7\,pc\ cm^{-6}$

Thus at $\tau \simeq 1$, if this turn-over is due to TBS, at $\tau \sim 1$ we can solve for

$$T_e \simeq 400K$$

This is way less than the brightness temp. near 10MHz → $T_B \simeq 10^7$ K at 20MHz. Thus, because T_B must always be less than the physical temp. of the emitting region, TBS cannot be the primary cause of this turn-over!

Practicing Synchrotron Calculations

a) A commonly used value to describe power-law radio spectra is the "spectral index" α, where this spectral index is defined as the slope of the spectrum in log-log space, thus $S \propto f^{\alpha}$. Show that if you measure flux densities S_1 and S_2 at two frequencies ν_1 and ν_2, the spectral index between these two frequencies can be written as:

$$\alpha = \frac{\log(S_1/S_2)}{\log(\nu_1/\nu_2)}$$

[Note: the spectral index is sometimes defined as $S \propto f^{-\alpha}$]

Set up as proportionality:

$$\frac{S_1}{S_2} = \left(\frac{f_1}{f_2}\right)^{\alpha}$$

$$\log\left(\frac{S_1}{S_2}\right) = \log\left[\left(\frac{f_1}{f_2}\right)^{\alpha}\right]$$

$$\log\left(\frac{S_1}{S_2}\right) = \alpha \log\left(\frac{f_1}{f_2}\right)$$

$$\alpha = \frac{\log\left(\frac{S_1}{S_2}\right)}{\log(f_1/f_2)}$$

b) Let's assume Cass A from the previous problem (note: read the previous problem) is effectively a sphere about 4 arcminutes in diameter that is filled with cosmic rays. The relativistic electrons emit synchrotron radiation with a spectrum

$$\frac{S}{\text{Jy}} = 2700\left(\frac{\nu}{\text{GHz}}\right)^{\alpha}$$

over the frequency range 10 MHz to 100 GHz, where $\alpha = -0.77$. What is the total luminosity (integrated luminosity, in solar units) of Cass A over this band?

Integrate $L_\nu d\nu = 4\pi D^2 S_\nu d\nu$ to get total luminosity, L.

$$L = 4\pi D^2 \int_{10\,\text{MHz}}^{100\,\text{GHz}} S_\nu d\nu \text{ , know } D = 3\,\text{kpc} = 9.3\times10^{19}\,\text{m}$$

$$\text{and } S_\nu = 2700\times10^{-26}\,\text{W m}^{-2}\,\text{Hz}^{-1}\left(\frac{\nu}{10^9\,\text{Hz}}\right)^{-0.77}$$

$$\hookrightarrow S_\nu = 2.3\times10^{-16}\,\text{W/m}^2\text{/Hz}\,(\nu/\text{Hz})^{-0.77}$$

$$L = 4\pi(9\times10^{19})^2 \times (2.3\times10^{-16}) \times \left(\frac{1}{0.23}\,\nu\right)\Big|_{10^7}^{10^{11}}$$

$$\hookrightarrow \boxed{L \simeq 3\times10^{28}\,\text{W} \simeq 84\,L_\odot}$$

c) Assuming that the relativistic electrons account for 3% of the cosmic ray energy density, what its the minimum magnetic field in Cass A? If you don't have sufficient equations in your lecture notes please refer to ERA! Also, please use the attached reference sheet for constant values. For the constants, base the spectral index on the previous problem.

3% e^- contribution: $\eta = \frac{U_i}{U_e} = \frac{100}{3} = 33$

Assume Equipartition $\left\{ B_{min} = 8\times10^{-5}\,\mathrm{T}\,\left([1+\eta]\,c_{12}\,L\right)^{2/7} R^{-6/7} \right.$

Source is 4' sphere $\rightarrow R = 3\,\mathrm{kpc}\cdot\sin(2') \simeq 3\,\mathrm{kpc}\cdot\frac{2}{60}\cdot\frac{\pi}{180} \simeq 5.4\times10^{16}\,\mathrm{m}$

Also need c_{12} for $\alpha = -0.77$ and $\nu_2 = 10^{11}\,\mathrm{Hz}$

$c_{12} \simeq 3.9\times10^{7}$ (from Pacholczyk 1970 handout) for $\alpha = -0.8$

$\hookrightarrow$ or $c_{12} \simeq 3.6\times10^{7}$ ~~[illegible]~~ interpolating for $\alpha = -0.77$

$B_{min} \simeq 7.3\times10^{-5}\,\mathrm{T} = 0.73\,\mathrm{mG}$

Much larger than Galactic diffuse field!!! ($\sim 10\,\mu\mathrm{G}$)

The functions:†

$$c_{12} = c_2^{-1} c_1^{1/2} \frac{2\alpha - 2}{2\alpha - 1} \cdot \frac{\nu_1^{(1-2\alpha)/2} - \nu_2^{(1-2\alpha)/2}}{\nu_1^{1-\alpha} - \nu_2^{1-\alpha}}$$

$$c_{13} = 0.921 \cdot c_{12}^{4/7}$$

for $\nu_1 = 10^7$ Hz and $\nu_2 = 10^{10}$ and 10^{11} Hz.

N.B. In these equations, $S_\nu \propto \nu^{-\alpha}$

α	$\nu_2 = 10^{10}$ Hz		$\nu_2 = 10^{11}$ Hz	
	c_{12}	c_{13}	c_{12}	c_{13}
0.2	2.5 E 07	1.6 E 04	8.3 E 06	8.3 E 03
0.3	2.8 E 07	1.7 E 04	9.8 E 06	9.1 E 03
0.4	3.2 E 07	1.8 E 04	1.2 E 07	1.0 E 04
0.5	3.7 E 07	2.0 E 04	1.6 E 07	1.2 E 04
0.6	4.5 E 07	2.2 E 04	2.0 E 07	1.4 E 04
0.7	5.4 E 07	2.5 E 04	2.8 E 07	1.7 E 04
0.8	6.5 E 07	2.7 E 04	3.9 E 07	2.0 E 04
0.9	7.8 E 07	3.0 E 04	5.4 E 07	2.4 E 04
1.0	9.3 E 07	3.3 E 04	7.1 E 07	2.8 E 04
1.1	1.1 E 08	3.6 E 04	9.3 E 07	3.3 E 04
1.2	1.3 E 08	4.0 E 04	1.1 E 08	3.7 E 04

† For $\alpha = 1/2$ and 1 the functions c_{12} and c_{13} have values following from the appropriate formulae resulting from the integration of equations (7.4) and (7.5).

Table 8 from Pacholczyk's Radio Astrophysics. *Here* $\nu_1 = \nu_{\min}$ *and* $\nu_2 = \nu_{\max}$.