ASTR469 Lecture 3: Magnitudes; Blackbodies (Still Ch. 5)

Assess yourself/study guide after lecture & reading (without peeking at notes)...

1. You observe a galaxy and determine that it is 2.3×10^4 times fainter than our Sun (that is, its flux is 2.3×10^4 less than that from the Sun). What is the absolute magnitude of that galaxy?

2. What is the apparent visual magnitude of a star with the same luminosity as Vega, but which is twice as far away?

3. Considering again the star from the previous problem, what is its absolute magnitude compared to that of Vega?

4. At what wavelength and frequency does the thermal emission from a typical human body peak? Note: humans are effectively blackbodies of a fixed temperature. Compare this with the thermal emission from a neutron star, whose temperatures are typically around 10^6 K?
(Interesting side note: Although the thermal emission from a neutron star's surface does not peak at radio wavelengths, some neutron stars are visible as pulsing objects in the radio because of a non-thermal emission process that sends streams of emission out of their magnetic poles!)

5. (Note: this one somewhat more complex than the previous problems.) Determine the $\lambda = 10\,\mu$m spectral brightness, flux density, and spectral luminosity you would observe from a human being standing at a distance of 15 m away from you. Assume for argument's sake that the human is a sphere of radius 1.1 m.

1 Magnitudes

Magnitudes are a way to quantify both observed and intrinsic light from an astronomical object (see below). They are typically only used in the optical and near-infrared regimes, and are always measured in a particular sub-band of those regimes. It is applied to the optical/IR light from anything seen at those wavelengths: including stars, galaxies, solar system objects, etc.

First a bit of history... magnitude scalings are ancient, and based on Hipparchus's classification of stars in the northern sky. Hipparchus was a Greek astronomer who classified stars with values of magnitude from 1 to 6, 1st magnitude being the brightest (excluding the Sun; he was considering only the night sky). Because it was defined by eye, and the eye does not have a linear response, a first magnitude star is not twice as bright as a second magnitude star. Instead, astronomers later found that Hipparchus' system is roughly logarithmic, and 6th magnitude stars are roughly 100 times fainter than 1st magnitude stars. The magnitude system has a few peculiarities:

1. It is defined backwards (bright things have lower, even negative, numbers).
2. It is logarithmic.

So using this system is a real hoot.

Five equal steps in log-space (1st to 6th magnitude) give factors of 2.512 in linear space ($100^{\Delta m/5} = 2.512^{\Delta m}$). Therefore, a magnitude 1 star is 2.512 times brighter than a magnitude 2 star, and a 4th magnitude star is $2.512^3 = 15.8$ times fainter than a 1st magnitude star.

We've now hard-wired these scalings in the magnitude system, which can range well beyond 1–6, but is still a **relative scale**; that is, magnitudes tell you *how intense some object appears (apparent magnitude) or how much luminosity some object has (absolute magnitude), compared to some other object.*

Apparent Magnitude

Apparent magnitudes are always written as a lower-case m. These tell you how bright some object appears compared to some other star:

$$m - m_{\rm ref} = -2.512 \log_{10}(F/F_{\rm ref}) \tag{1}$$

or

$$\frac{F}{F_{\rm ref}} = 10^{0.4(m_{\rm ref}-m)}\,, \tag{2}$$

where F and $F_{\rm ref}$ are the fluxes, and m and $m_{\rm ref}$ are the magnitudes. Because this is a relative scale, obviously we need a reference object of known flux and magnitude. Any star will do, but two commonly used ones are Vega and the Sun. The Sun has an apparent visual magnitude of $m_V = -26.74$, whereas Vega is $m_V = +0.03$.

Absolute Magnitude

We can also talk about "absolute" magnitudes, which quantify the actual energy output of the object. We use capital "M" for absolute magnitudes. The difference is that apparent magnitudes are to flux as absolute magnitudes are to luminosities, thus it follows that:

$$\frac{L}{L_{\text{ref}}} = 10^{0.4(M_{\text{ref}}-M)} \tag{3}$$

The Sun has an absolute visual magnitude of $M_V = 4.8$.

Because magnitudes are unitless, we can directly compare absolute and apparent magnitudes, and relate this to distance. This is also defined somewhat arbitrarily, relating the two by the flux that an object would have at a distance of 10 pc:

$$\frac{F_{10\,\text{pc}}}{F} = 10^{\frac{m-M}{2.5}} = \left(\frac{d}{10\,\text{pc}}\right)^2 \tag{4}$$

Or more simply:

$$m - M = 5\log d - 5\,, \tag{5}$$

where d is the distance to the source *in units of parsec*. We see from this equation that the absolute and apparent magnitudes are the same when $d = 10\,\text{pc}$.

Brief Aside: Photometric Filters

Instead of arbitrary wavelengths, we usually use photometric filters. The most common optical filters used are the Johnson *U,B,V,R,I*, but there are now a large number of filters available. Magnitudes found using these filters are often denoted with the filter names themselves, e.g., B for m_B. We can also have absolute magnitudes denoted as M_B.

Each filter can be characterized as having a central wavelength and a width. During observations, you must specify which filter you are interested in using. Here are some common visual filters, called "Johnson" filters:

Filter Letter	**Effective Midpoint**	**Full Width Half Maximum**	**Description**
U	365 nm	66 nm	"U" stands for ultraviolet.
B	445 nm	94 nm	"B" stands for blue.
V	551 nm	88 nm	"V" stands for visual.
R	658 nm	138 nm	"R" stands for red.
I	806 nm	149 nm	"I" stands for infrared.

2 Optical Depth and the Effects of Intervening Media on a Spectrum

Intervening Media

Remember that before we were saying that intensity is conserved in free space; that is, the emission and observation intensity would be the same as long as there was nothing influencing it along the way. Let's think just a little bit about what might happen along the way. Generally there are three things: Something might be absorbing your light, scattering your light (both of these are collectively "attenuation"), or there might be some other emitting object along your line of sight. Prolific examples of intervening media in space: dust, neutral gas, ionized gas (plasma).

Kirchoff's Laws

The spectrum you observe depends on the density (the optical depth) of the object, and the viewing direction. Observing the same object from a different direction may give you a different signal. **Kirchoff's Laws** tell us how to interpret the spectra we observe. Kirchoff actually coined the term "blackbody radiation" and has a ton of Laws in a few areas of physics, so it is safe to assume that Kirchoff was smart and interesting at parties.

Kirchoff's three laws of spectra are:

- A dense object produces light with a continuous (blackbody) spectrum.
- A hot diffuse gas produces an emission line spectrum due to electronic transitions within the gas. Fluorescent lights are a good example.
- A hot dense object seen through a cooler gas (i.e., cooler than the hot object) makes you see absorption lines. The absorption lines are at exactly the same wavelengths as the emission lines for a given element or molecule, and are also due to electronic transitions.

Astronomers have a term that characterizes how transparent some medium appears at a certain wavelength: the optical depth, τ_ν. The "optical" here doesn't refer only to optical light, it just means generically "how well we can observe through the medium with τ_ν at frequency ν." Here are some general truths about this quantity:

- It goes from zero to infinity, zero meaning no attenuation (transparent, like a clean window for optical light), infinity meaning complete absorption (completely opaque, like a wall for optical light).
- If optically thick, you are not seeing the entire emission from the source (like a door; you only see the top layer of atoms because it is optically thick).

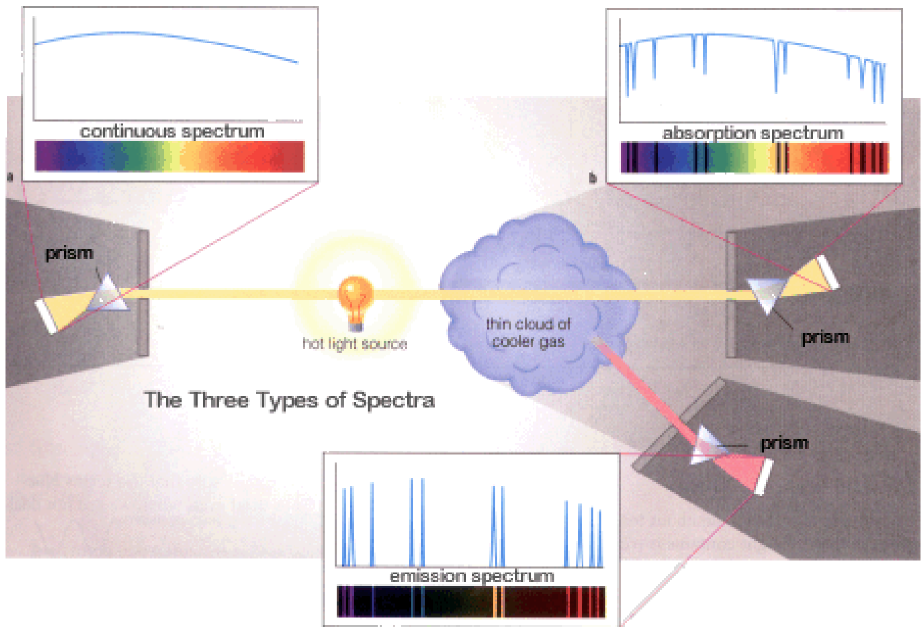

Figure 1: Types of spectra demonstrating Kirchoff's laws of spectra. (The emission spectrum is observing the cloud itself, while the absorption spectrum is observing the actual light source influenced by the cloud. The "continuous spectrum" shows what the light source would look like unattenuated.)

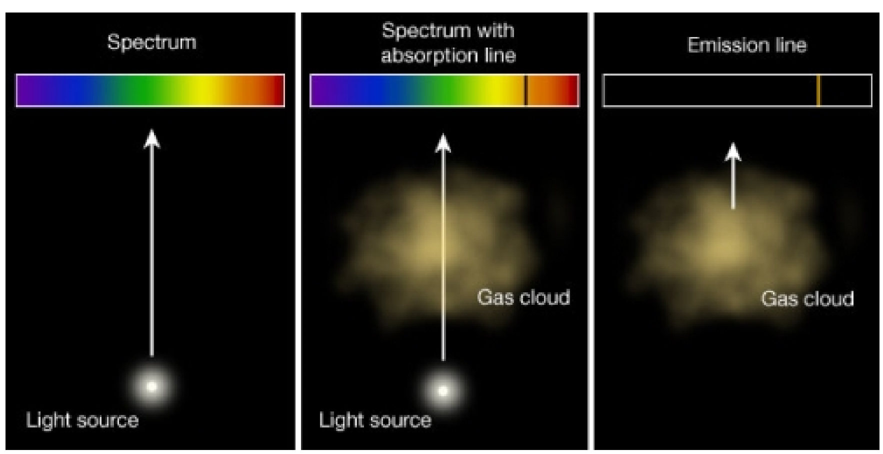

Figure 2: Uninterrupted light, light absorbed by a cool(er) cloud, and emission line observed from a cloud itself.

- If a source is optically thick at a particular wavelength, it appears as a blackbody!
- At certain frequencies a region may become optically thin (for instance, radio signals go through doors; X-rays go through skin but not bones). So if I observed the door in the radio spectrum, I will not see it as a blackbody. The transparency of some medium depends on a number of factors, including the particle density, the speed of particles, and the magnetic fields involved.

Equation of Radiative Transfer

Note: you won't be tested on EORT or required to know it for this course, but I think it may help you to know a little bit about where τ_ν comes from. The EORT is an official phrase that just describes "how some intervening medium effects your observation of a source by its absorption and emission." The medium might itself be emitting some light at your observing frequency, or it might be absorbing some of the light at your observing frequency (it might also scatter your light, but the equation below ignores this). If a source emits with intensity I_ν, in some little part of the medium ds, it might absorb a little bit of that intensity, dI_ν:

$$dI_\nu = -\kappa_\nu I_\nu ds + j_\nu ds \tag{6}$$

Where:

- κ is the absorption coefficient, telling you how absorptive the medium is.
- j is the emission coefficient, telling you how much light the medium itself creates.

There is a huge field of research aimed at solving this equation to determine how I_ν is changed by a medium under particular conditions for κ and j, but for our sake, we only really care about astronomical media. Almost all of these media (like stellar atmospheres, interstellar plasma clouds, or our own atmosphere), are fairly homogenous and are at a fixed single temperature (i. e. they're in local thermodynamic equilibrium). In that case, the observed intensity is:

$$I_\nu \simeq (1 - e^{-\tau_\nu}) B_\nu(T) + I_0 \, e^{-\tau_\nu} \ , \tag{7}$$

where B_ν is the Planck function and I_0 is the radiation going into the medium. You can see that if the optical depth goes to infinity, we only see the Planck function: that is, **optically thick objects appear as blackbodies**. If the optical depth is zero, we only see the background radiation (no light attenuation, like a window).

3 Blackbody Emission

Blackbody radiation is pretty important: many stars emit approximately as blackbodies, and anything at a temperature $T > 0\,\mathrm{K}$ will have some blackbody emission at part of the EM band, as long as it's sufficiently dense.

You all should be familiar with blackbodies from Modern Phys (PHYS314), where you would have seen the Planck functions:

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}\,. \tag{8}$$

or as a function of wavelength:

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}\,. \tag{9}$$

where k is the Boltzmann constant, $k = 1.38\times10^{-23}\,\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$.

But, what you may not have recognized are the units: $\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}$ for B_ν, or alternately $\mathrm{W\,m^{-2}\,m^{-1}\,sr^{-1}}$ for B_λ, where the additional "m^{-1}" term accounts for the wavelength (often given in meters, Angstroms, cm). Does this look familiar? It should! It is a spectral intensity! So this gives the spectral intensity of a blackbody.

So, under what conditions is $I_\nu = B_\nu$? Or in other words, when does the spectral intensity distribution look like a blackbody? The object must be dense and optically thick. Stars get close enough. Gas clouds do not, since they're very tenuous at all wavelengths.

Let's look at some fun things we can do with the Planck function.

1. **At what wavelength or frequency does a blackbody spectrum peak?** To do this we need to find the maximum of the function by taking the derivative of B and setting it equal to zero: $dB_\lambda/d\lambda = 0$ or $dB_\nu/d\nu = 0$. This gives us the **Wien displacement law**:

$$\lambda_{\max} = \frac{2.898\times10^{-3}}{T}\ \mathrm{m}\,, \tag{10}$$

or

$$\nu_{\max} = 5.879\times10^{10}\,T\,. \tag{11}$$

Note that temperature T here (and usually throughout astronomy) always has units of Kelvin. This tells us that hotter things peak at shorter wavelengths (higher frequencies)! As we'll see next class, this gives us a nice direct, physical link between the colors and temperatures of stars.

2. **We can make the Planck function easier to use at very low frequencies.** We can do this by recognizing that at those low frequencies, $h\nu \ll kT$, and expanding out the Planck function. This gives us the **Rayleigh-Jeans approximation**:

$$B_\lambda(T) = \frac{2ckT}{\lambda^4} \tag{12}$$

or

$$B_\nu(T) = \frac{2kT\nu^2}{c^2} \tag{13}$$

As you will explore in your homework, this approximates the Planck function well at low frequencies (long wavelenths).

3. **How can we get the *total* intensity instead of just the spectral intensity of a blackbody?** As before, to get total intensity we want to integrate over the whole EM band.

$$B(T) = \int_0^\infty B_\nu(T) d\nu \tag{14}$$

If you do the math you'll find that $B(T) = \frac{\sigma T^4}{\pi}$, where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. In the case of an isotropic radiation field, which we can almost always assume, it can be shown that $F_{\mathrm{surf}} = \pi I$, where F_{surf} is the flux coming out of the surface of an object. So therefore:

$$F_{\mathrm{surf}} = \sigma T^4 \tag{15}$$

This is of course the **Stephan-Boltzmann Law**. We are often interested in the total luminosity of an object (in erg s^{-1} or W), for which we integrate the flux over the total surface the light is emitted from: $L = \int_S F_{\mathrm{surf}} dA$, getting:

$$L = A\sigma T^4 \tag{16}$$

which you saw as a sneak preview in last class's lecture notes! For spherical objects, this leads to $L = 4\pi r^2 \sigma T^4$, where r is the object's radius. Thus, the total energy output is related to the surface area and the temperature. Note that this is the same luminosity as given last lecture, except that this one is appropriate when you know the emergent flux from the surface of an object, and the other one is appropriate when you know the observed flux that you measure with your telescope. Both are fluxes and have the same units, but just have slightly different contextual meanings.