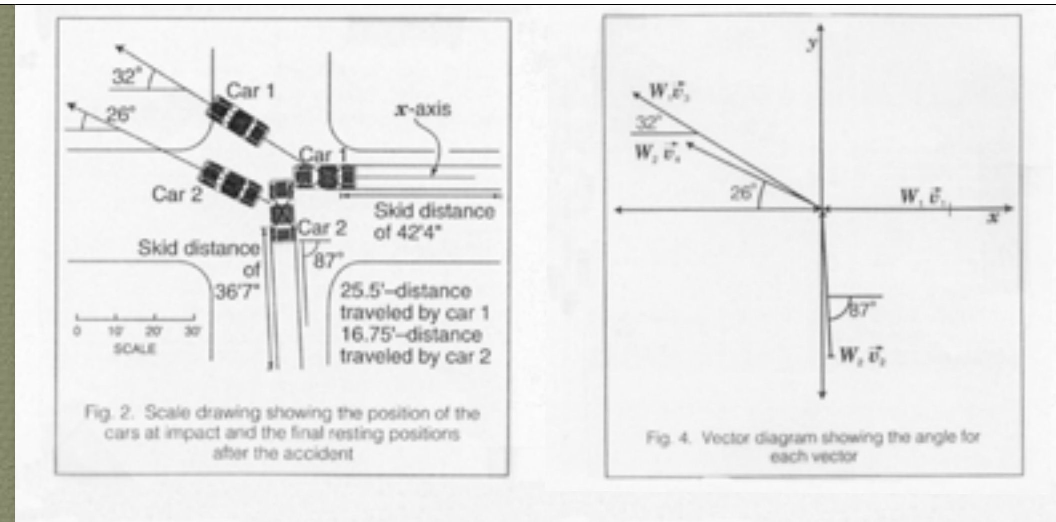


Collisions in Two Dimensions

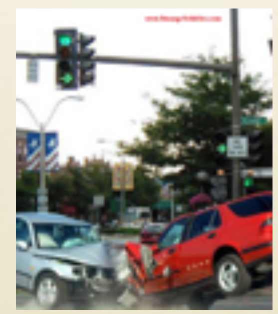
Why physicists are so awesome at pool, and How to reconstruct car accidents!

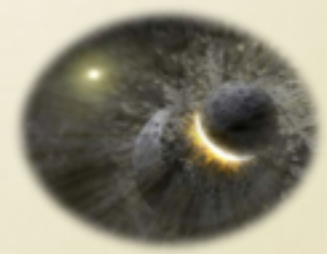
Practice: 6.31, 6.33, 6.39, 6.41, 6.43, 6.45, 6.47, 6.49, 6.51, 6.53, 6.55, 6.63, 6.73

I hope you all were inspired last night by the Oscars to work on your physics movie assignment!

Main Ideas in Class Today

- **You should be able to:**
    - Distinguish between Elastic and Inelastic Collisions
    - Solve Collisions in 1 & 2 Dimensions



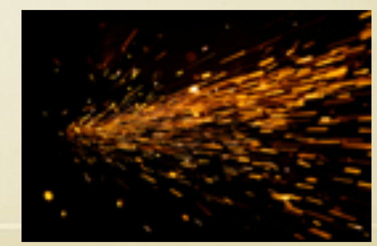


Reminders

Momentum is always conserved.
(in isolated systems)

"Mechanical energy"
is the sum of kinetic and potential energy.

Energy is always conserved
(but not always mechanical energy!)



Energy losses to sound, lots to heat, and sometimes light.

How do we know what direction and speed things will go after a collision?

Has it bothered anyone that in our discussion of conservation of momentum we have two velocities.masses going in and two coming out, but no obvious way to determine a priori how both of these balls will act after the collision? I’m here to tell you today that any collision can be modelled if we account for all major factors of energy transfer during the collision. I.e. not just how the momentum is transferred between objects, but where does the energy go? This is the basis of forensic car crash investigations and lots of crash-related insurance claims, and we’ll explore that.

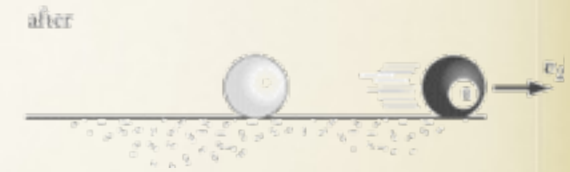
How do we know what direction and speed things will go after a collision?

Today we'll consider both
MOMENTUM and **ENERGY**!

Has it bothered anyone that in our discussion of conservation of momentum we have two velocities.masses going in and two coming out, but no obvious way to determine a priori how both of these balls will act after the collision? I'm here to tell you today that any collision can be modelled if we account for all major factors of energy transfer during the collision. I.e. not just how the momentum is transferred between objects, but where does the energy go? This is the basis of forensic car crash investigations and lots of crash-related insurance claims, and we'll explore that.

Collisions: A Range of Momentum and Energy Exchange.

Elastic



Momentum and kinetic energy conserved

***The idea is this…Elastic: happens to RIGID OBJECTS. Billiard balls are a fair example of this.

***Most collisions are somewhere along the range of collision types, but some can be approximated as elastic. For instance in billiard balls, there is some energy lost to heat of their collision.

***Perfectly inelastic is perhaps the easiest to solve, as you only get one velocity afterwards [pf=vf(m1+m2)].

Collisions: A Range of Momentum and Energy Exchange.

Elastic

Momentum and kinetic energy conserved

Inelastic

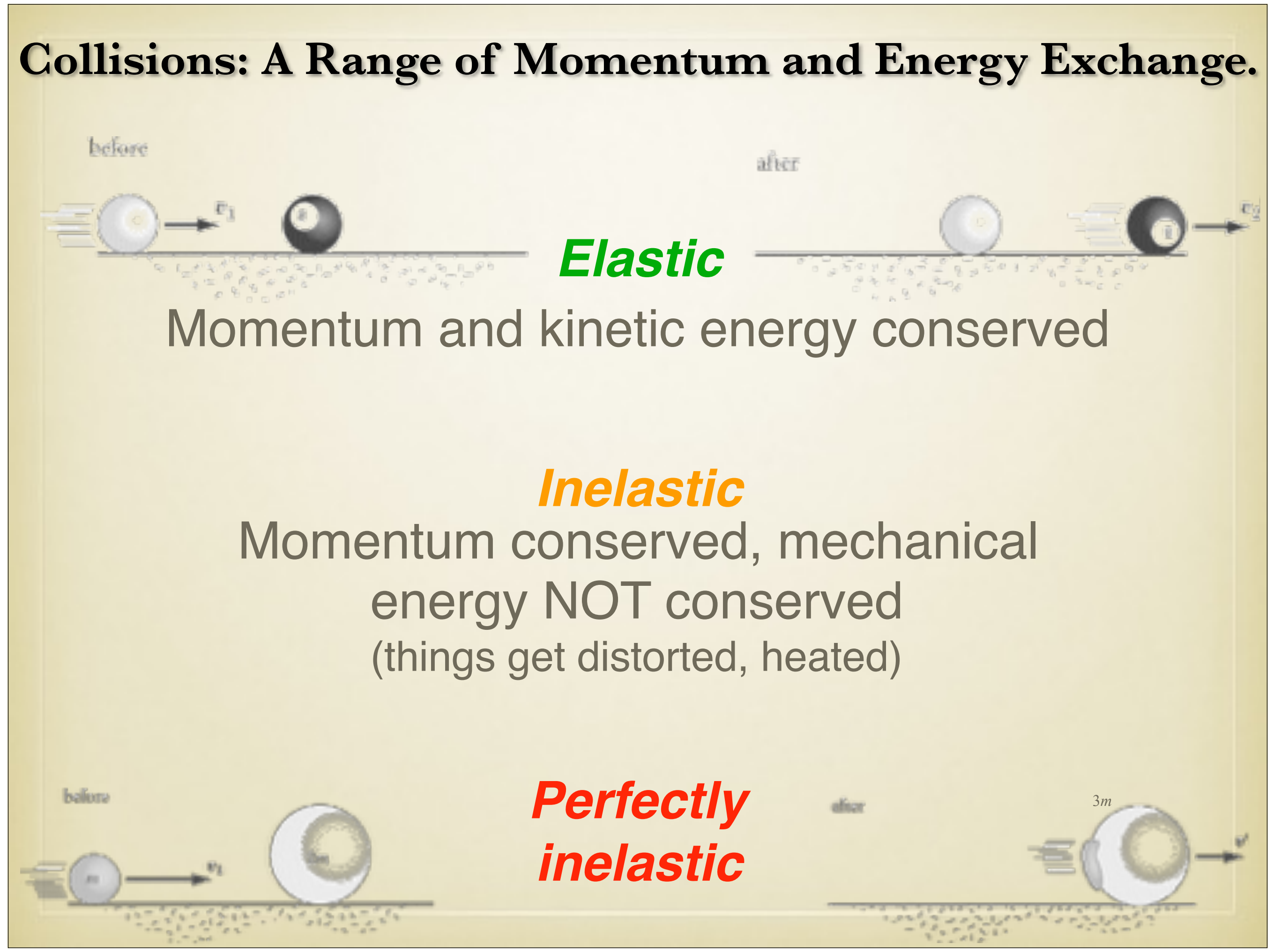
Momentum conserved, mechanical energy NOT conserved

(things get distorted, heated)

***The idea is this…Elastic: happens to RIGID OBJECTS. Billiard balls are a fair example of this.

***Most collisions are somewhere along the range of collision types, but some can be approximated as elastic. For instance in billiard balls, there is some energy lost to heat of their collision.

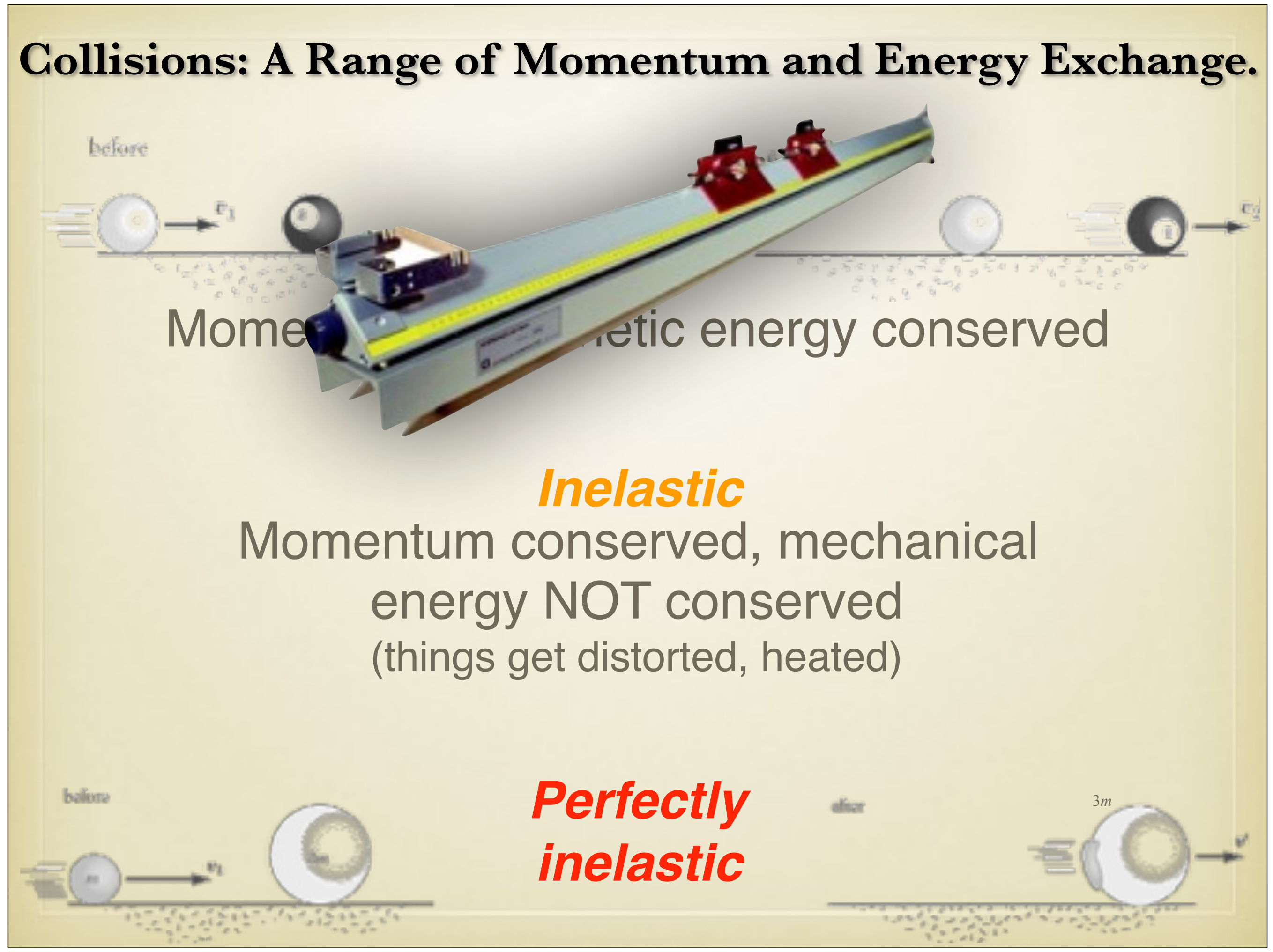
***Perfectly inelastic is perhaps the easiest to solve, as you only get one velocity afterwards [pf=vf(m1+m2)].



***The idea is this…Elastic: happens to RIGID OBJECTS. Billiard balls are a fair example of this.

***Most collisions are somewhere along the range of collision types, but some can be approximated as elastic. For instance in billiard balls, there is some energy lost to heat of their collision.

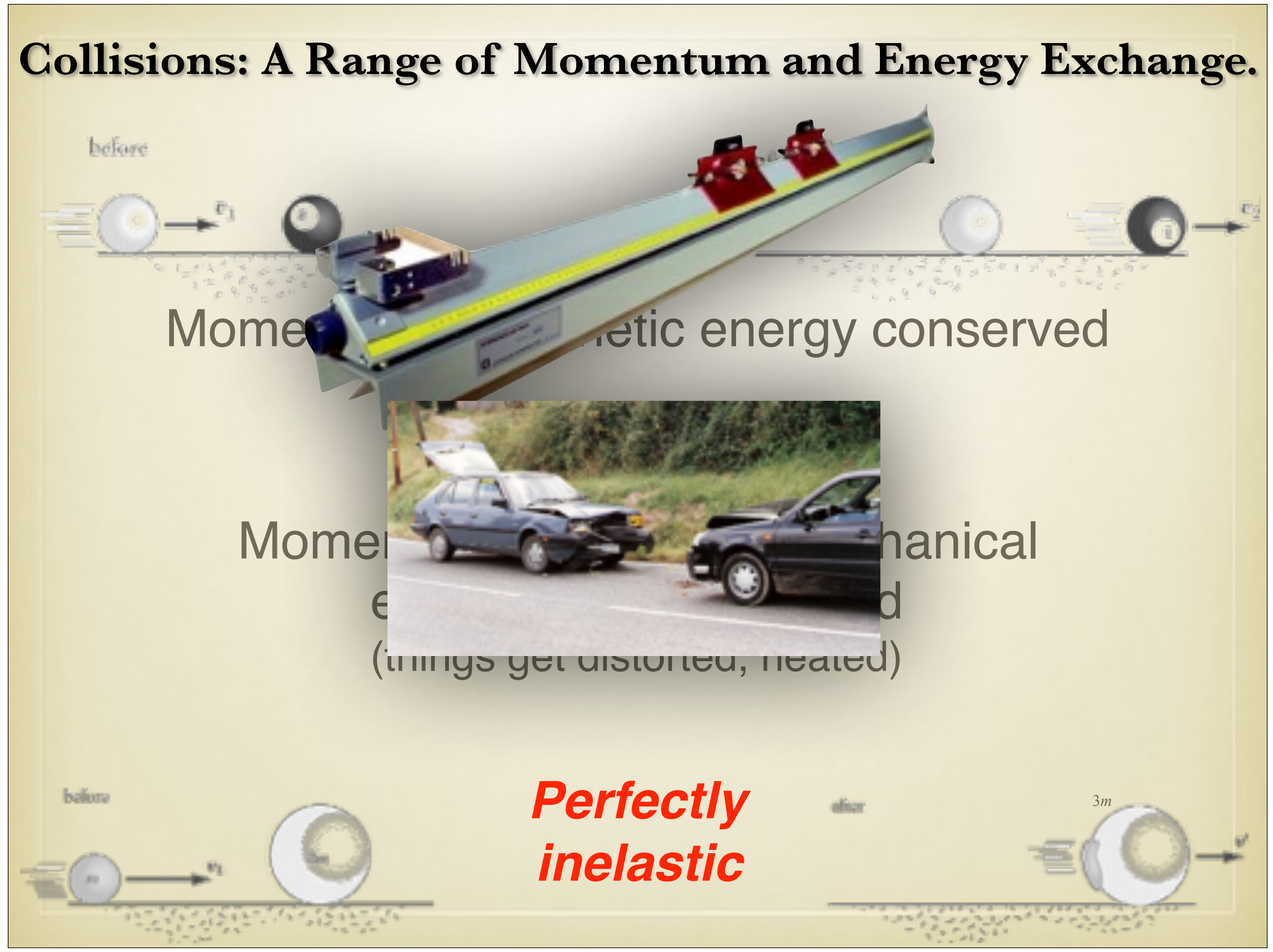
***Perfectly inelastic is perhaps the easiest to solve, as you only get one velocity afterwards [pf=vf(m1+m2)].



***The idea is this…Elastic: happens to RIGID OBJECTS. Billiard balls are a fair example of this.

***Most collisions are somewhere along the range of collision types, but some can be approximated as elastic. For instance in billiard balls, there is some energy lost to heat of their collision.

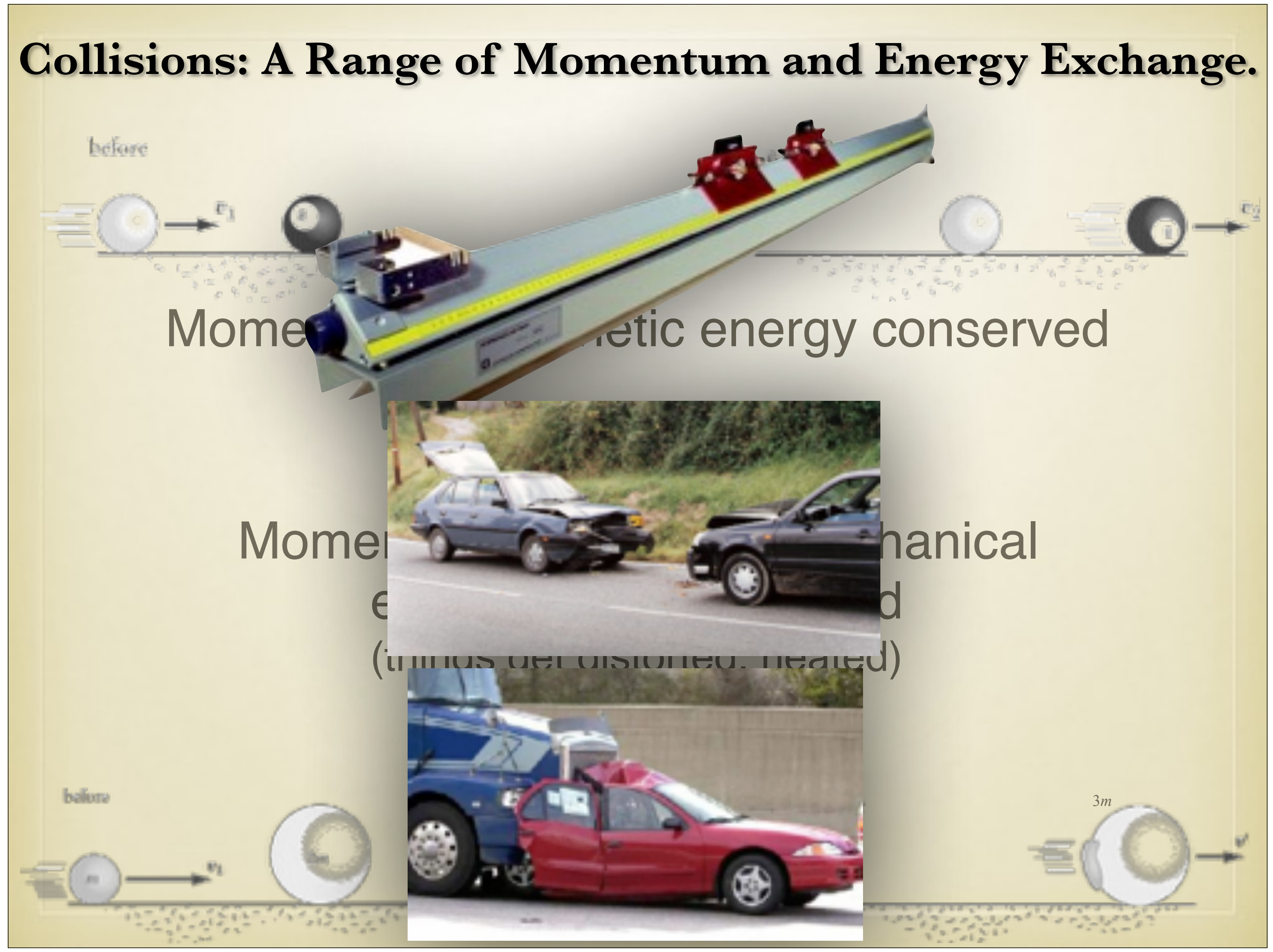
***Perfectly inelastic is perhaps the easiest to solve, as you only get one velocity afterwards [pf=vf(m1+m2)].



***The idea is this…Elastic: happens to RIGID OBJECTS. Billiard balls are a fair example of this.

***Most collisions are somewhere along the range of collision types, but some can be approximated as elastic. For instance in billiard balls, there is some energy lost to heat of their collision.

***Perfectly inelastic is perhaps the easiest to solve, as you only get one velocity afterwards [pf=vf(m1+m2)].



***The idea is this…Elastic: happens to RIGID OBJECTS. Billiard balls are a fair example of this.

***Most collisions are somewhere along the range of collision types, but some can be approximated as elastic. For instance in billiard balls, there is some energy lost to heat of their collision.

***Perfectly inelastic is perhaps the easiest to solve, as you only get one velocity afterwards [pf=vf(m1+m2)].

For both **elastic** and **inelastic** collisions linear momentum is conserved (unlike energy)

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Only for **elastic collisions**, mechanical energy (KE+PE) of the system is also conserved.
If not a vertical collision, then $PE_i = PE_f$ and

$$KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$$

deltaPE always can be treated as (at least close to) 0 since just before and just after collision (even if has y component).

Different form of Kinetic energy equation where actually energy exchange no longer depends on mass in an elastic collision. Your RELATIVE velocities are the same but in the opposite direction. So you can imagine from your vantage point it looks like the car is coming toward you at some speed and then going away from you with the same speed. SAY THIS TO YOURSELF TWICE and let it sink in, then look back at the equation! This is true for an elastic collision, but not an inelastic one.

For both **elastic** and **inelastic** collisions linear momentum is conserved (unlike energy)

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Only for **elastic collisions**, mechanical energy (KE+PE) of the system is also conserved.
If not a vertical collision, then $PE_i = PE_f$ and

$$KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$$

Thus… (see sec 6.3, book):

$$(\vec{v}_1 - \vec{v}_2)_i = -(\vec{v}_1 - \vec{v}_2)_f$$

deltaPE always can be treated as (at least close to) 0 since just before and just after collision (even if has y component).

Different form of Kinetic energy equation where actually energy exchange no longer depends on mass in an elastic collision. Your RELATIVE velocities are the same but in the opposite direction. So you can imagine from your vantage point it looks like the car is coming toward you at some speed and then going away from you with the same speed. SAY THIS TO YOURSELF TWICE and let it sink in, then look back at the equation! This is true for an elastic collision, but not an inelastic one.

Elastic collision example.

Suppose that a 2000.0-kg car, initially at rest, is struck head on by a 36,000.0-kg semitruck moving at 20.0 m/s. Determine the velocity of each of the vehicles after the collision, **assuming that the collision is elastic**.

Use conservation of momentum

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

and then apply conservation of energy.

$$(\vec{v}_1 - \vec{v}_2)_i = -(\vec{v}_1 - \vec{v}_2)_f$$

Solve system of equations.

BOTH momentum and energy are conserved in an elastic collision.
Is elastic a good assumption for this collision? No, bumpers are there for a reason!

Elastic collision example.

Suppose that a 2000.0-kg car, initially at rest, is struck head on by a 36,000.0-kg semitruck moving at 20.0 m/s. Determine the velocity of each of the vehicles after the collision, **assuming that the collision is elastic**.

(This is not a very good assumption because it would assume they bounce right off of each other instead of crushing the bumpers/car.)

Perfectly inelastic collision example.

Suppose that a 2000.0-kg car, initially at rest, is struck head on by a 36,000.0-kg semitruck moving at 20.0 m/s. Determine the velocity of each of the vehicles after the collision, **assuming that the vehicles stick together**.

Use conservation of momentum

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

…noting that there is only one final velocity!

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = \vec{v}_f (m_1 + m_2)$$

NOW I'm going to tie in concepts that we've learned about previously, to help your conceptual thinking about the various types of collisions. Going to be some clicker questions.

If you had to pick one, which car would you rather be in, for a truck/car head on collision (assume you want to minimize your injuries)?

A. The car during the elastic collision

B. The car during the inelastic collision

C. Doesn't matter, experience same force either way

Q61

Impulse with short time = bad! Think about FΔt for each of these.
Answer: B

A compact car and a large truck collide head on and stick together. Which vehicle undergoes the larger magnitude of acceleration during the collision?

A. car
B. truck
C. Both experience the same acceleration.
D. Can't tell without knowing the initial
E. and final velocities.

Q62

HINT IF NEEDED: Think about F=ma. If you're checking this by solving equations forget momentum for a moment and consider newton's 3rd and 2nd laws.

Answer: A.

Newton's 3rd law says same force. Acceleration different due to Newton's second law.

[Shown on light board]

The car has inertial mass m and the truck has inertial mass $M \gg m$. Because the changes in momentum are equal (neglecting the fact that they are in opposite directions), we have $m\Delta v = M\Delta V$, where Δv is the change in the car's speed and ΔV the change in the truck's speed. Because $m \ll M$, $\Delta v \gg \Delta V$. The acceleration is proportional to the change in speed, and both changes in speed take place over the same time interval (the duration of the collision). Therefore the car undergoes a much larger acceleration than the truck.

A compact car and a large truck collide head on and stick together. Which vehicle undergoes the larger magnitude of acceleration during the collision?

Why don't you need the velocities?

A. car
B. truck
C. Both experience the same acceleration.
D. Can't tell without knowing the initial
E. and final velocities.

Q62

HINT IF NEEDED: Think about F=ma. If you're checking this by solving equations forget momentum for a moment and consider newton's 3rd and 2nd laws.

Answer: A.

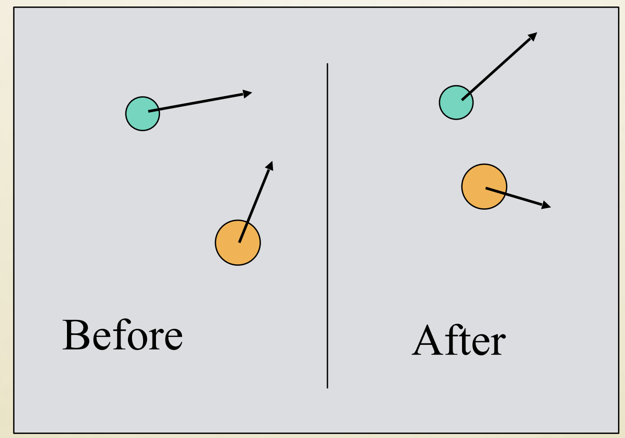
Newton's 3rd law says same force. Acceleration different due to Newton's second law.

[Shown on light board]

The car has inertial mass m and the truck has inertial mass $M >> m$. Because the changes in momentum are equal (neglecting the fact that they are in opposite directions), we have $m\Delta v = M\Delta V$, where Δv is the change in the car's speed and ΔV the change in the truck's speed. Because $m << M$, $\Delta v >> \Delta V$. The acceleration is proportional to the change in speed, and both changes in speed take place over the same time interval (the duration of the collision). Therefore the car undergoes a much larger acceleration than the truck.

Collisions and Conservation in Two Dimensions

$$\Sigma \boldsymbol{p_i} = \Sigma \boldsymbol{p_f}$$



If collision is **elastic**, then we also still have $KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$

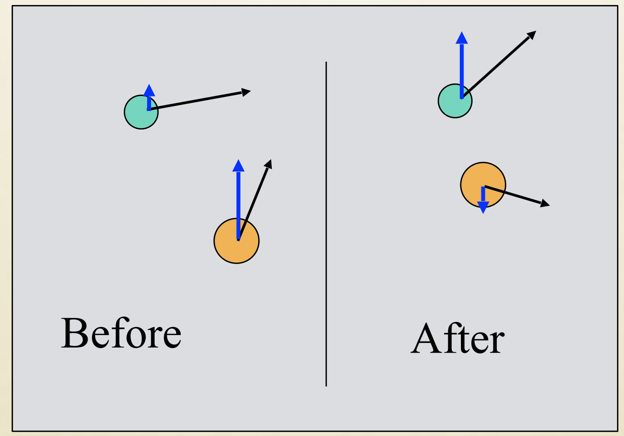
We keep saying that momentum and velocity are vectors so we can break them down into components. Now we'll do this.

If you get really good at this stuff you'll be able to beat me at pool.

Collisions and Conservation in Two Dimensions

$$\Sigma p_i = \Sigma p_f$$

$$\Sigma p_{iy} = \Sigma p_{fy}$$



If collision is **elastic**, then we also still have $KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$

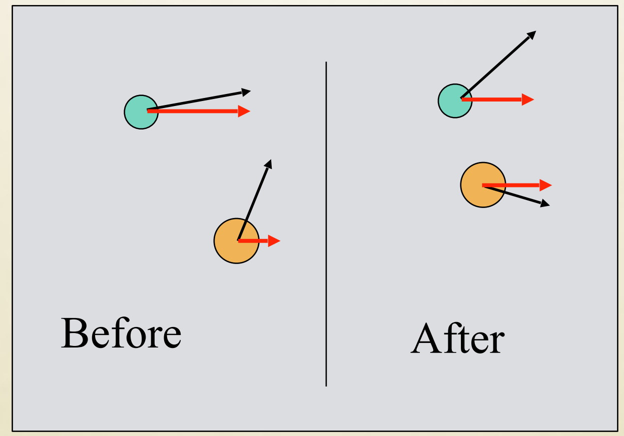
We keep saying that momentum and velocity are vectors so we can break them down into components. Now we'll do this.

If you get really good at this stuff you'll be able to beat me at pool.

Collisions and Conservation in Two Dimensions

$$\Sigma p_i = \Sigma p_f$$

$$\Sigma p_{ix} = \Sigma p_{fx}$$



If collision is **elastic**, then we also still have $KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$

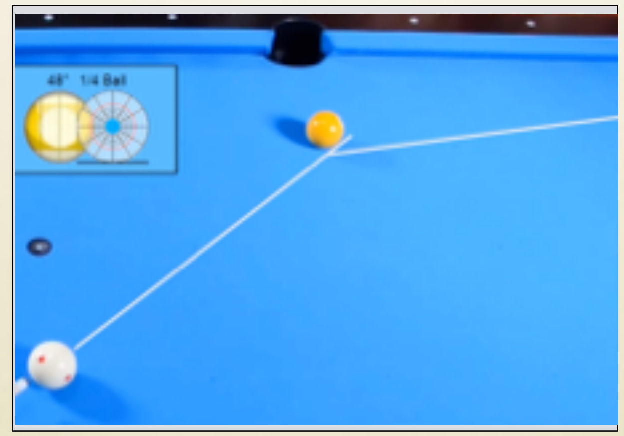
We keep saying that momentum and velocity are vectors so we can break them down into components. Now we'll do this.

If you get really good at this stuff you'll be able to beat me at pool.

Collisions and Conservation in Two Dimensions

$$\Sigma p_i = \Sigma p_f$$

$$\Sigma p_{iy} = \Sigma p_{fy} \qquad \Sigma p_{ix} = \Sigma p_{fx}$$



If collision is **elastic**, then we also still have $KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$

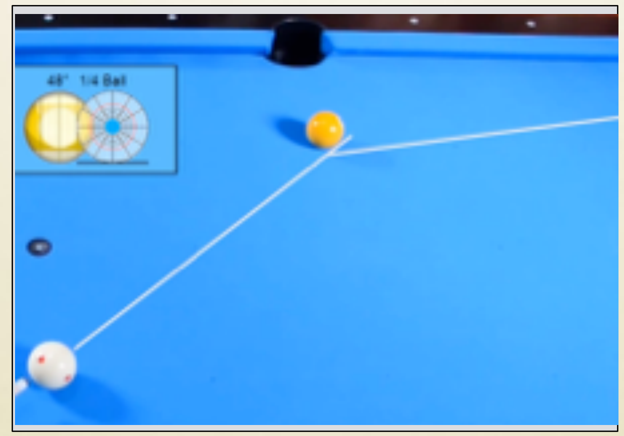
We keep saying that momentum and velocity are vectors so we can break them down into components. Now we'll do this.

If you get really good at this stuff you'll be able to beat me at pool.

Collisions and Conservation in Two Dimensions

$$\Sigma p_i = \Sigma p_f$$

$$\Sigma p_{iy} = \Sigma p_{fy} \qquad \Sigma p_{ix} = \Sigma p_{fx}$$



If collision is **elastic**, then we also still have $KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$

We keep saying that momentum and velocity are vectors so we can break them down into components. Now we'll do this.

If you get really good at this stuff you'll be able to beat me at pool.

Prepare yourself for mystery and excitement.

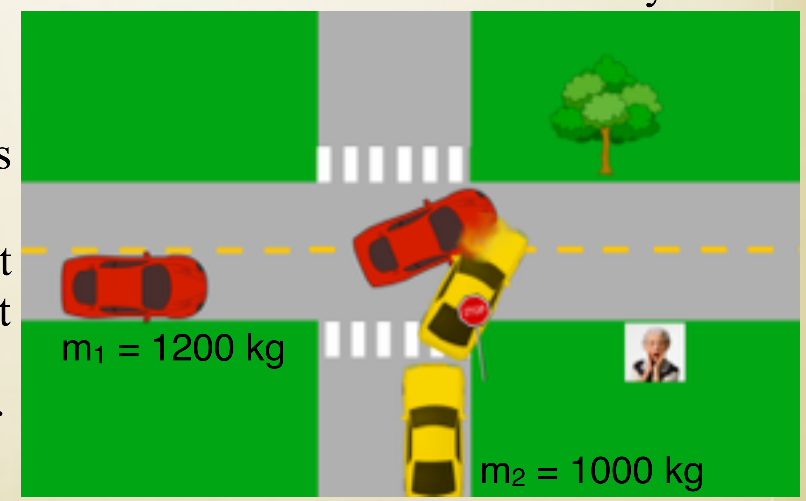
Prepare yourself for mystery and excitement.

There is a collision at an intersection and the police take statements.

Driver car 1: “I was minding my own business, totally driving the speed limit (40 mph) and this jerk just ignored the stop sign and plowed through at full speed (25 mph speed limit)!!!”

Driver car 2: “This guy has it out for me. I was stopped at the stop sign, and the driver slowed almost to a stop and waved me on, so I eased out into the intersection. Then he accelerated like crazy!”

Elderly bystander: “I looked when I heard the cars collide. They were jammed together and travelling about 25 mph straight towards that tree! I was so scared.” [other observers tell similar stories]



We can set this up as a problem of verifying Car Driver 1’s story against those of innocent bystanders. If one car was driving 40mph east and one was driving 25mph north, what is the cars’ final velocity and angle of trajectory?
[Done on light board].