Radio Astronomy (ASTR700) Problem Set #2
Radiation fundamentals and Fourier transforms

To ensure you obtain full credit, explain all of your work; don’t just write equations and numbers. Tell me (briefly) what you’re doing and why. Try all parts of a problem, even if you can’t solve earlier parts.

Each question part is worth 10 points unless otherwise stated [total: 80/80].

1. The “habitable zone” around a star is defined as the region in which liquid water can exist on a planet. Let’s figure out what this is for our solar system. Assume that the Earth is an isothermal blackbody in radiative equilibrium with solar heating: that is, all the solar power absorbed by Earth’s projected surface goes into heating Earth, and is reradiated as blackbody radiation at some fixed temperature $T_E$.

(a) Derive an expression of the temperature of Earth as a function of distance from the Sun (note, do this symbolically; however as a self-check, if you plug in approximately correct values for your variables, you should arrive at a temperature of around 280 K).

(b) Estimate the inner and outer radii, $r_{in}$ and $r_{out}$, of the habitable zone around the Sun by calculating the distance from the Sun at which the radiative equilibrium temperature of an isothermal black body is the boiling temperature of water ($\sim 373$ K) and the distance at which water freezes ($\sim 273$ K). Express $r_{in}$ and $r_{out}$ in units of astronomical units (AU), where 1 AU is defined as the radius of the Earth’s orbit around the Sun.

(c) (5 points) At a wavelength of 68 cm, Jupiter was found to have a brightness temperature of more than 500 K. Could the observed temperature of Jupiter be caused by solar heating (note: you may need to look up some size/distance parameters for Jupiter)? Discuss your answer briefly.

\textit{Note:} “isothermal” here means that the whole planet is the same temperature regardless of day-side/night-side. To be actually nearly isothermal, an actual planet or asteroid would have to be spinning fast enough and/or have enough atmosphere that its daytime and nighttime temperatures are about equal).

2. \textbf{This problem is meant to help you consider how to treat different noise sources.} On a crisp morning in Green Bank, during your observations the actual thermometer temperature of the atmosphere is 275 K. You are observing at 5 GHz at zenith. Figure 1 on the next page is reproduced from ERA for your convenience. Explain your answers.

(a) What is the noise power coming from the atmosphere? Lest you be led astray, recall that the atmosphere is not a perfect blackbody.\footnote{Note that we alluded to this in class but did not directly show it. By way of hint, consider the solution we have seen for an isotropic blackbody with non-zero opacity. What emission should the atmosphere itself create based on this equation? If you get stuck, you could also read up on the relevant material in ERA section 2.2.}

(b) What is the noise power coming from a receiver cooled such that it has a noise temperature of 20 K?
The opacity of the Earth’s atmosphere causes two deleterious effects: it weakens astronomical signals received on the ground due to its absorption (signal absorption is proportional to $e^{-\tau}$, as mentioned in our first lecture and discussed in ERA). On top of this, the atmosphere also adds to the system noise due to its emission. Both effects directly degrade the signal-to-noise ratio $S/N$ of ground-based observations because $S/N$ is directly proportional to the received flux density of the source and inversely proportional to the system noise temperature, $T_{\text{sys}}$. As in ERA section 2.2.3, we can write the system noise temperature as $T_{\text{sys}} = T^{*}_{\text{sys}} + T^{*}_{\text{atm}}$, where $T^{*}_{\text{sys}}$ includes all contributions to the system temperature except the contribution from observing through the atmosphere, $T^{*}_{\text{atm}}$. Based on $T^{*}_{\text{sys}} = 20 \text{ K}$, which has a greater effect in lowering the $S/N$, atmospheric absorption or atmospheric emission?

3. (15 points) Derive the modulation theorem (ERA Equation A.12 and given in class). The modulation theorem is the basis of the ubiquitous “superheterodyne receiver” (ERA Section 3.6.4 and Figure 3.39). Let the variable $x$ be time $t$, so the complementary variable $s$ is frequency, $\nu$. In a superheterodyne receiver, the input radio-frequency signal $f(t)$ is multiplied in a mixer by a monochromatic local oscillator signal $\cos(2\pi \nu_{\text{LO}} t)$ at frequency $\nu_{\text{LO}}$. The result is that input radio-frequency signals at frequency $\nu_{\text{RF}}$ are shifted in frequency to the intermediate frequencies $\nu_{\text{IF}} = \nu_{\text{RF}} - \nu_{\text{LO}}$ and $\nu_{\text{IF}} = \nu_{\text{RF}} + \nu_{\text{LO}}$. For example, a local oscillator at $\nu_{\text{LO}} = 12 \text{ GHz}$ can be used to shift an input radio signal at frequency $\nu_{\text{RF}} = 9 \text{ GHz}$ down to a lower frequency $\nu_{\text{IF}} = 3 \text{ GHz}$ where it is easier to process. Hint: Rewrite $\cos(2\pi \nu x)$ as a complex exponential; ERA Appendix B.3 may be a useful reference.

4. To discover pulsars, one usually collects a series of time samples and looking at their power spectrum (via Fourier transform of the time series) for high-amplitude peaks. Let’s pretend you’re searching for pulsars with the Very Large Array, with relatively coarse time sampling; you’re collecting time samples that each have a duration of 1 ms. What is the highest-frequency pulsar you could, in principle, be able to discover in this data?